

k-Distributions and Solar Heating

Topics:

1. k-distributions
2. Correlated k-distributions
3. Solar heating
4. Chapman profile

Reading: Liou 4.3; Thomas 10.4,10.5.4

Problems with band models:

- Curtis-Godson approximation for inhomogeneous paths is not accurate when absorber and pressure are not correlated (e.g. O_3).
- Band models can't handle scattering - they assume source function is constant, but scattering source function depends on radiance.
- A heating rate calculation with band models goes as N^2 where N is the number of layers.

The k-distribution Method

Band mean transmission is

$$\mathcal{T}_{\Delta\nu}(u) = \frac{1}{\Delta\nu} \int_{\Delta\nu} e^{-k_\nu u} d\nu$$

The order in which the wavenumbers are summed doesn't matter!

So sum them from low k to high k .

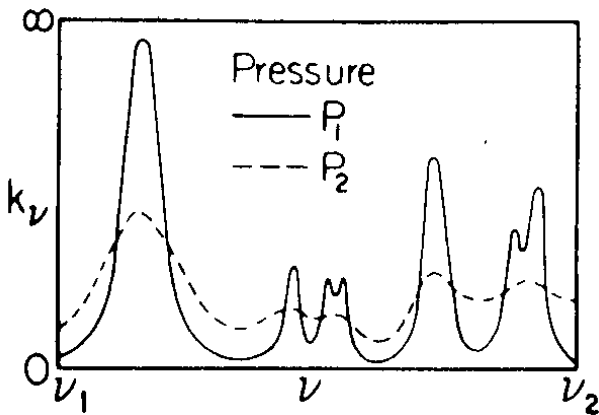
Rearrange the integral to be over the absorption coefficient k :

$$\mathcal{T}_{\Delta\nu}(u) = \frac{1}{\Delta\nu} \int_{\Delta\nu} e^{-k_\nu u} d\nu = \int_0^\infty e^{-ku} f(k) dk$$

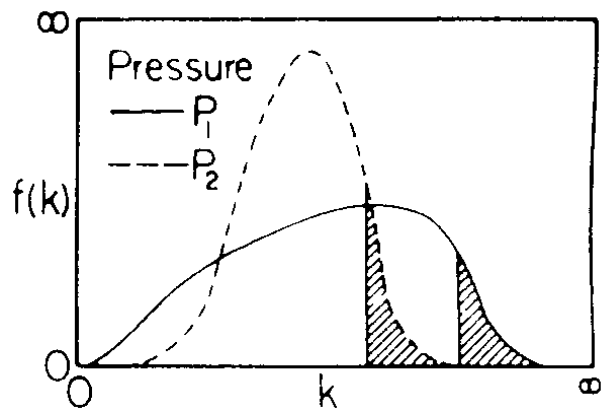
$f(k)$ is the fraction of the spectral band with absorption $k \rightarrow k + dk$.

$f(k)$ is the probability density function of absorption, $\int f(k) dk = 1$.

$f(k)$ is smooth \rightarrow few terms in integration sum.



(a) Absorption coefficient $k(\nu)$



(b) Probability distribution $f(k)$

A schematic illustration portraying the essence of the k-distribution method. (a) A schematic of absorption line spectra at two different pressures. (b) The two probability density functions $f(k)$ associated with (a). The shaded area depicts the stongest absorption (i.e. largest k 's) which come from the same spectra regions. [from G. L. Stephens]

Cumulative Distribution Function

It is simpler to integrate over the cumulative probability function

$$g(k) = \int_0^k f(k) dk$$

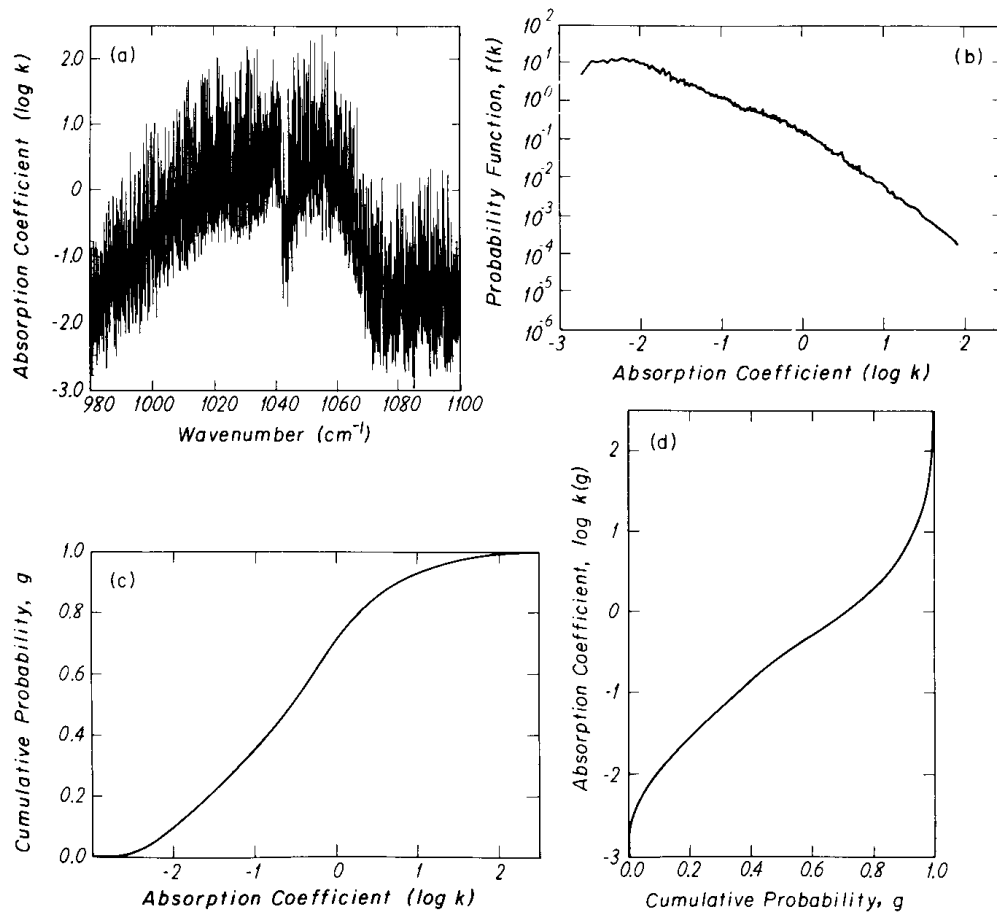
$g(k)$ is the fraction of the spectrum with absorption coef. below k .

One-to-one correspondence between k and g :

$g = 0$ is the lowest k , $g = 1$ is the highest k .

Band mean transmission is

$$\mathcal{T}_{\Delta\nu}(u) = \int_0^1 e^{-k(g)u} dg$$



(a) Absorption coefficient k_ν in units of $\text{cm}^{-1} \text{atm}^{-1}$ as a function of wavenumber with a resolution of 0.05 cm^{-1} in the $9.6 \mu\text{m}$ ozone band ($p = 30 \text{ mb}$; $T = 200 \text{ K}$). (b) The probability function $f(k)$ of the absorption coefficient. (c) The cumulative probability distribution function of $f(k)$ plotted as a function of k . (d) Same as (c), except that values of the absorption coefficient are expressed as a function of g . [Liou (1992), Fig. 2.15]

Discrete k-distribution Sums

Discrete sum is accurate because $k(g)$ is smooth. Divide g range from 0 to 1 up into intervals Δg_j over which $k(g)$ can be approximated to be constant:

$$\mathcal{T}_{\Delta\nu}(u) \approx \sum_{j=1}^N e^{-k_j u} \Delta g_j$$

Δg_j is the probability interval: fraction of spectrum represented by the mean absorption coefficient k_j .

One “ k ” in sum corresponds to Beer’s law: monochromatic, gray (no spectral variation), or low optical depth.

Making a k-distribution

- 1) Laplace transform transmission $T(u)$ (simple for some band models).
- 2) Exponential sum fitting: $k_j, \Delta g_j$ from fitting transmission $T(u)$.
Use line-by-line model to compute mean transmission, non-linear least squares to fit $T(u)$ as a function of u .
- 3) Compute $g(k)$ directly from line-by-line k_ν spectrum.
Sort k_ν , start of array is $g = 0$, end (highest k) is $g = 1$.
Divide sorted array into Δg_j intervals, average each to get k_j .

Inhomogeneous Paths - Correlated k-distribution

Each pressure and temperature along a path has a unique k_ν spectrum.

A correlated k-distribution sorts each $k_\nu(p, T)$ spectrum independently to make k-distributions $k(g; p, T)$ for each p and T .

In practice, discrete k-distributions $k_j(p_l, T_m)$ are made for a set of pressures p_l and temperatures T_m and interpolated in between.

Use the k_j ’s for all L layers as in monochromatic calculations:

$$\mathcal{T}_{\Delta\nu} = \sum_{j=1}^N \Delta g_j \exp \left(- \sum_{l=1}^L k_j(z_l) \Delta u_l \right)$$

Correlated k-distribution Justification

Absorption spectra for a gas at different pressures are correlated.

Correlated k assumption is exact when each k-distribution $k(g; p, T)$ sorts the wavenumbers in exactly the same order, e.g. a single line or evenly spaced lines. CKD also gives exact transmission for the weak line and strong line limits.

In reality, sorting all layer spectra on total optical depth does not give monotonic absorption coefficient vs. cumulative probability.

The real justification is that correlated k errors are $< 1\%$ in typical atmospheric radiative transfer computations.

Using a k-distribution

Do a weighted sum of monochromatic calculations (one for each “ k ”):

1. For each layer l interpolate k-distribution absorption coefficient to layer pressure and temperature to get $k_j(z_l)$.

2. Multiply $k_j(z_l)$ by absorber amount in layer and add to particle scattering optical depth in layer: $\Delta\tau_j(z_l) = k_j(z_l)\Delta u(z_l) + \tau_{sca}(z_l)$

3. Solve monochromatic radiative transfer equation using the layer optical depths to get radiances or fluxes (e.g. $F_j(z_l)$). Use the band integrated Planck function or solar flux for the source function.

4. Weight result for each k in sum by Δg_j : $F_{\Delta\nu}(z_l) = \sum_j \Delta g_j F_j(z_l)$ to get the spectral band integrated flux profile.

Overlap of Gases in Spectral Band

What do we do about multiple gases absorbing in one spectral band?

One method: assume absorption spectra are independent. Then

$$\int_{\Delta\nu} (\mathcal{T}_\nu^{(1)} - \bar{\mathcal{T}}^{(1)}) (\mathcal{T}_\nu^{(2)} - \bar{\mathcal{T}}^{(2)}) d\nu = 0$$
$$\int_{\Delta\nu} \mathcal{T}_\nu^{(1)} \mathcal{T}_\nu^{(2)} d\nu = \bar{\mathcal{T}}^{(1)} \bar{\mathcal{T}}^{(2)} = \sum_{m=1}^M \Delta g_{1m} e^{-k_{1m}u_1} \sum_{n=1}^N \Delta g_{2n} e^{-k_{2n}u_2}$$
$$\bar{\mathcal{T}}(u_1, u_2) = \sum_{n=1}^N \sum_{m=1}^M \Delta g_{1m} \Delta g_{2n} e^{-\tau_{mn}} \quad \tau_{mn} = k_{1m}u_1 + k_{2n}u_2$$

$N \times M$ terms is a problem.

Another method treats ratio of absorber amounts, $\eta = u_2/u_1$, as another parameter like p , and tabulates k-distributions in u_1 as a function of η (i.e. $k_1(g; p, T, \eta)$).

An example correlated k-distribution

Fu and K. N. Liou, 1992: On the correlated k-distribution method for radiative transfer in nonhomogeneous atmospheres, *J. Atmos. Sci.*, 49, 2139-2156.

Divides shortwave into 6 bands with total of 54 k's and longwave into 12 bands with total of 121 k's.

Band	Region (cm ⁻¹)	<i>N</i> g's	Gases	Solar Flux (W/m ²)
1	50000-14500	10	O3	619.62
2	14500-7700	8	H2O	484.30
3	7700-5250	12	H2O	149.85
4	5250-4000	7	H2O	48.73
5	4000-2850	12	H2O	31.66
6	2850-2500	5	H2O	5.80
7	2200-1900	2	H2O	Overlap method
8	1900-1700	3	H2O	
9	1700-1400	4	H2O	
10	1400-1250	4	H2O, CH4, N2O	1
11	1250-1100	3	H2O, CH4, N2O	1
12	1100- 980	5	H2O, O3	1
13	980- 800	2	H2O	
14	800- 670	10	H2O, CO2	2
15	670- 540	12	H2O, CO2	2
16	540- 400	7	H2O	
17	400- 280	7	H2O	
18	280- 0	8	H2O	

A more accurate k-distribution for Earth's atmosphere is now available from the LBLRTM group at AER (Mlawer et al., 1997, Radiative Transfer for Inhomogeneous Atmospheres: RRTM, a Validated Correlated-k Model for the Longwave, *JGR*, 102, 16663-16682). It uses 16 k's in each band, and has 16 bands in the longwave and 14 bands in the shortwave. RRTM includes more gases, is based on updated line parameters, and comes with the latest continuum absorption model.

Solar Heating Rates

Net flux convergence causes heating

$$\left. \frac{dT}{dt} \right|_{rad} = -\frac{1}{\rho C_p} \frac{dF_{net}}{dz} = \frac{g}{C_p} \frac{dF_{net}}{dp}$$

In shortwave, no emission means no cooling, only heating from absorption.

In general, solar flux solution requires scattering, but Beer's Law on solar beam is a reasonable approximation for clear sky heating rates. (There is Rayleigh scattering in UV, but this occurs below where most UV is absorbed).

Spectrally integrate Beer's Law

$$F \downarrow = \mu_0 \int F_{\odot, \nu} \exp(-\tau_{\nu}/\mu_0) d\nu$$

μ_0 is cosine solar zenith angle, $F_{\odot, \nu}$ is TOA solar flux.

Solar Flux Convergence is flux lost from beam by absorption:

$$\frac{dF_{\nu} \downarrow}{dz} = \int F_{\odot, \nu} \beta_{\nu} \exp(-\tau_{\nu}/\mu_0) d\nu$$

where β is volume extinction coefficient ($\beta = -\tau/dz$). Heating rate is independent of μ_0 for $\tau \ll 1$.

Chapman Profile

Assume absorption k_{ν} is constant with height, density ρ is exponential.

$$\beta_{\nu} = k_{\nu} \rho_0 \exp(-z/H)$$

Integrate to get profile of monochromatic solar flux convergence:

$$\frac{dF \downarrow}{dz} = F_{\odot} \beta e^{-\tau/\mu_0} = F_{\odot} k \rho_0 e^{-z/H} \exp \left[-\frac{k \rho_0 H}{\mu_0} e^{-z/H} \right]$$

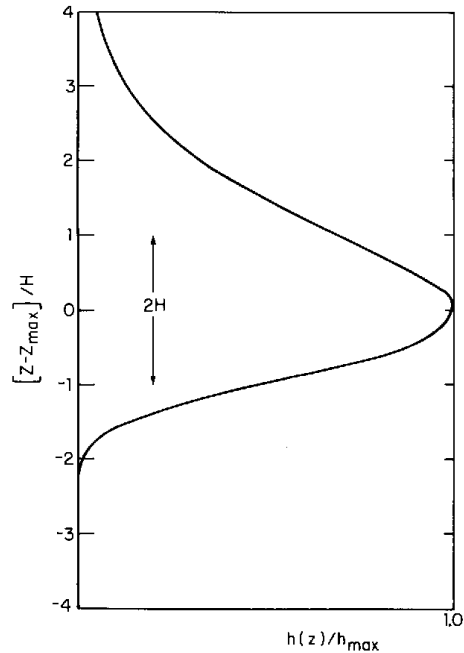
Peak net flux convergence:

$$\left(\frac{dF \downarrow}{dz} \right)_{\max} = \frac{F_{\odot} \mu_0}{eH} \quad \tau_{\max} = \mu_0 \quad z_{\max} = H \ln(k \rho_0 H / \mu_0) \quad \rho_{\max} = \frac{\mu_0}{kH}$$

Relevant for well mixed gases, e.g. O₂ in short ultraviolet.

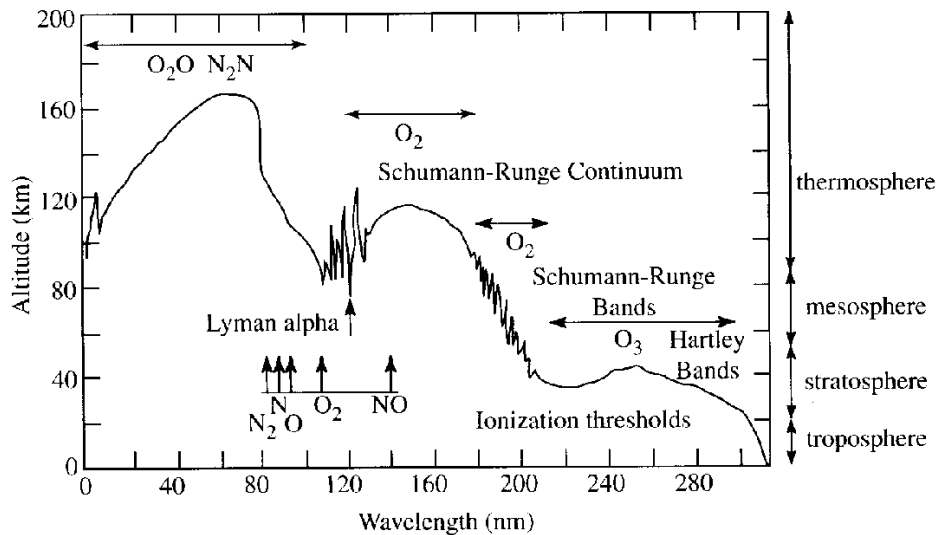
Higher absorption coefficient \rightarrow higher z_{\max} (lower ρ_{\max}).

Most monochromatic flux convergence in layer about 2 scale heights H . For a range of absorption coefficients (range of wavelengths) the layer is wider.



A Chapman layer where the solar energy is absorbed. The altitude is scaled by the absorber scale height H and the net flux convergence $h(z)$ is normalized to the maximum value. [Goody and Yung, Fig. 6.9]

Shorter UV wavelengths have more absorption and hence solar radiation is deposited at higher altitudes.

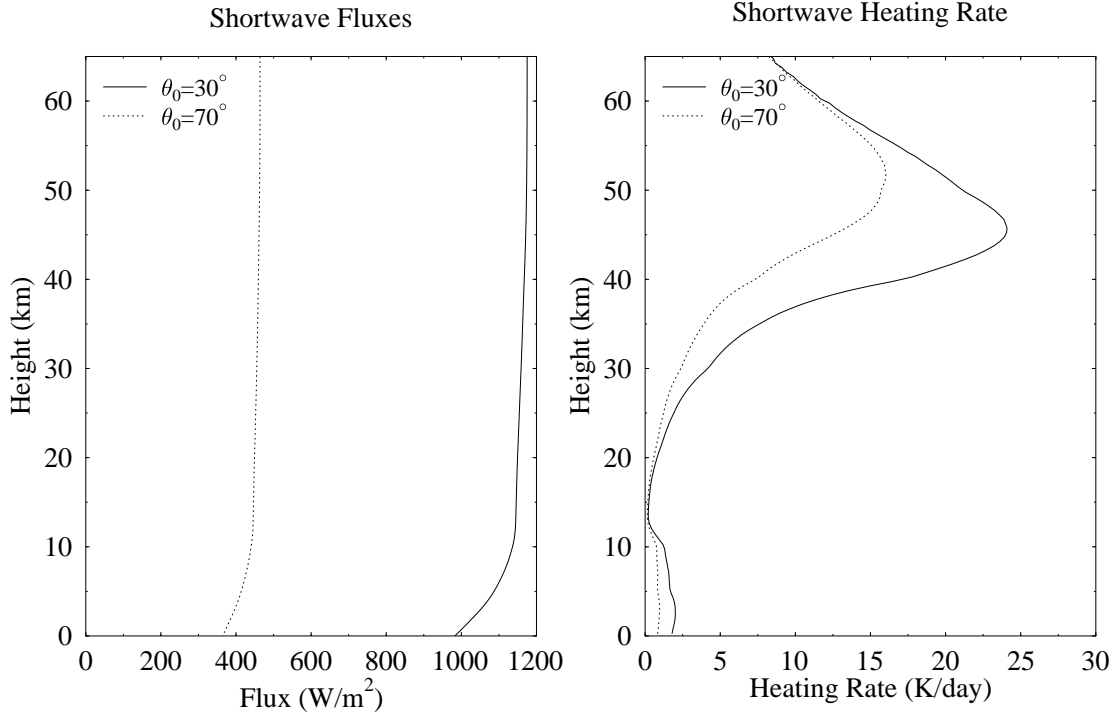


Atmospheric penetration depth versus wavelength. Horizontal arrows indicate the molecule (and band) responsible for absorption in that spectral region. Vertical arrows indicate the ionization threshold of the various species. [Thomas and Stamnes, Fig. 9.3]

Solar Broadband Fluxes and Heating Rates

Calculated with Fu and Liou (1992) k-distribution (6 solar bands, total of 54 k's) assuming no scattering and a black surface.

Fu RT profiles for midlatitude summer atmosphere



Fu RT profiles for midlatitude winter atmosphere

