Week 5: September 24-28

Band Transmission and Heating Rates

Topics:
1. Band transmission
2. Band models
3. Weighting functions
4. Heating rate
5. Broadband fluxes and heating rate profiles

Reading: Liou 4.4,4.7; Thomas 5.7,10.1-10.3,11.2

Band Transmission

The high spectral detail of molecular absorption lines ($\sim 0.001\ \mathrm{cm}^{-1}$) prevents rapid radiative transfer computations across the spectrum (line-by-line models are very slow).
Approximate radiative transfer methods divide the spectrum into spectral bands from 10 to 100 cm^{-1} wide. In each band the Planck function is approximately constant. Longwave radiative transfer can then be formulated in terms of mean transmission between levels.
Hence the need for the **mean transmission across a spectral band**: $\mathcal{T}_{\Delta\nu}$

- *Band models* give mean transmission for absorber amount u.
- *k-distributions* are a newer, more flexible method for spectrally averaged radiative transfer.

Single Line Transmission

The effect of many absorption lines on band transmission can be understood by first considering the spectrally integrated transmission of a single line.
Average absorption across a single line is

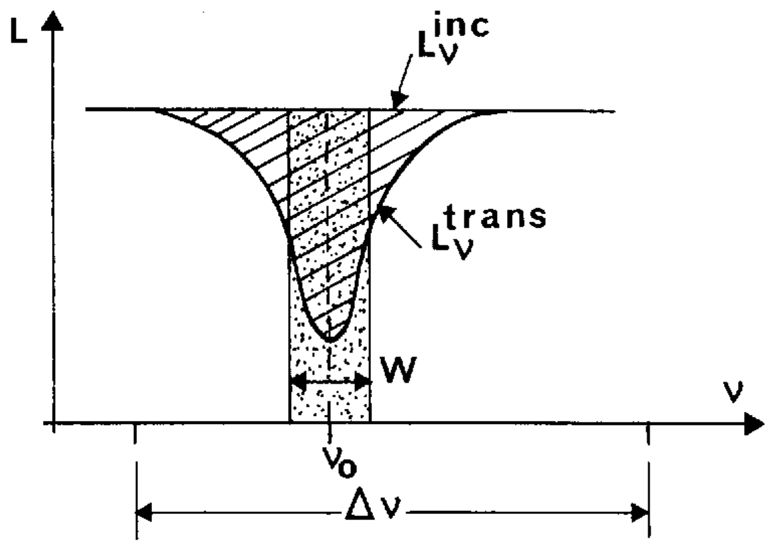
$$A_{\Delta\nu}(u) = 1 - \mathcal{T}_{\Delta\nu} = \frac{1}{\Delta\nu}\int_{\Delta\nu}(1 - e^{-k_\nu u})d\nu$$

Problem with average absorption is that it depends on $\Delta\nu$.

Equivalent Width

$$W = \Delta\nu A_{\Delta\nu} \; = \int_{\Delta\nu} (1 - e^{-k_\nu u}) d\nu$$

has units of spectral interval (cm^{-1}),
is the equivalent width of a fully absorbing ($A = 1$) rectangular line.



Schematic diagram illustrating the equivalent width. The dotted rectangular area is equal to the hatched area and represents the total energy absorbed in the line. [Lenoble, Fig. 8.2]

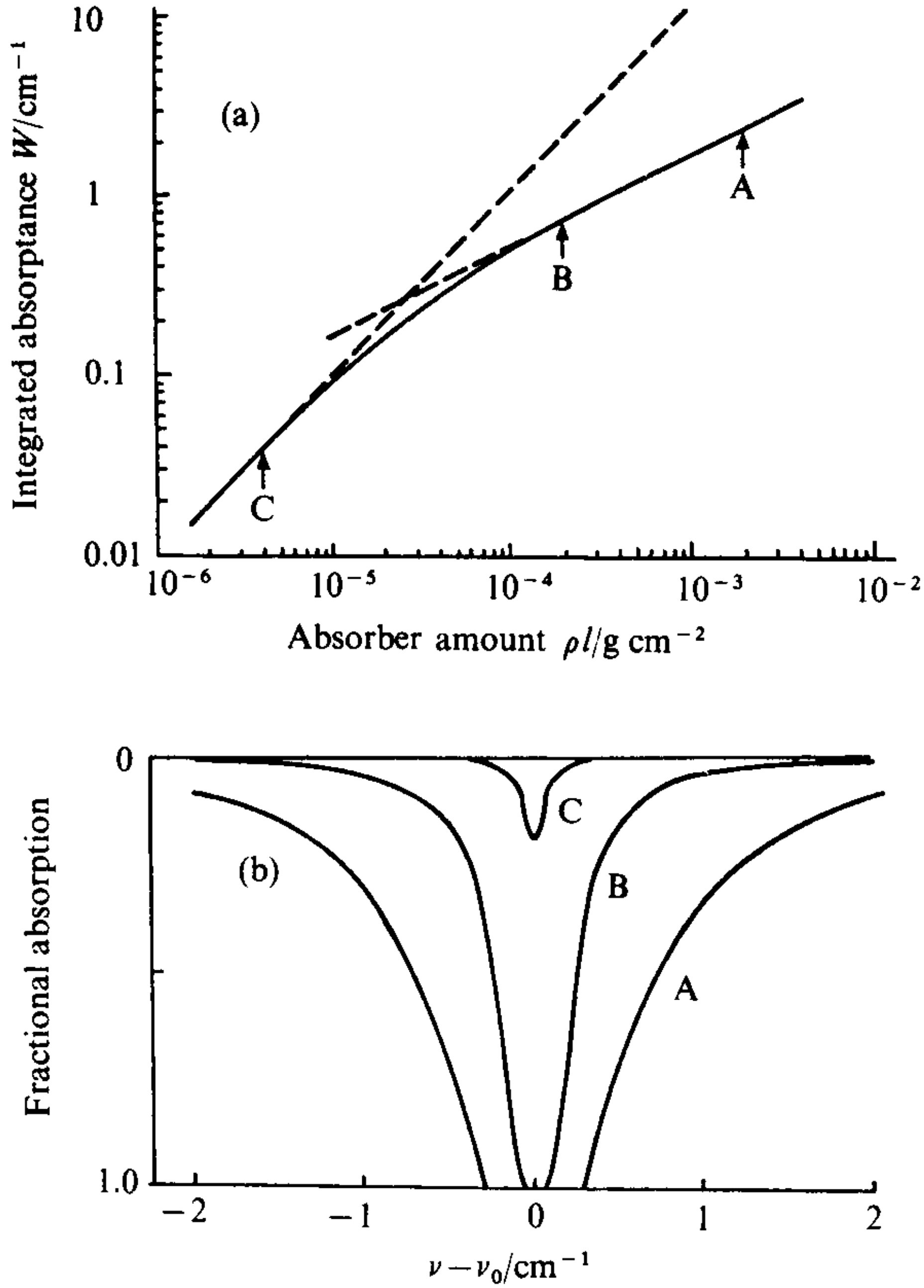
Equivalent Width of Lorentz Line

$$W = \int \left[1 - \exp\left(\frac{-Su\alpha/\pi}{(\nu - \nu_0)^2 + \alpha^2} \right) \right] d\nu \;\; = \; 2\pi\alpha L(x) \qquad x = \frac{Su}{2\pi\alpha}$$

$L(x) = xe^{-x}[I_0(x) + I_1(x)]$ is the *Ladenburg and Reiche* function.
(I_0 and I_1 are modified Bessel functions.)
A useful approximation is $L(x) \approx x[1 + (\pi x/2)^{5/4}]^{-2/5}$.

Weak line limit: $k_\nu u \ll 1 \quad W \approx \int k_\nu u d\nu \; = Su$
Linear in absorber amount and line strength.
Strong line limit: $k_\nu u \gg 1 \quad W = 2\sqrt{Su\alpha}$
Line center saturates, absorption increases from expanding width.

Curve of growth describes increase in absorption with absorber amount.



(a) Curve of growth of a typical spectral line with $S = 10^4\ \mathrm{cm}^{-1}\ (\mathrm{g\ cm}^{-2})^{-1}$ and $\alpha_0 = 0.06\ \mathrm{cm}^{-1}$ showing the linear and square root regions of growth. (b) Actual shapes of the transmission spectrum for different values of absorber amount $u = \rho l$ corresponding to the values shown in the arrows in (a). [Houghton, Fig. 4.2]

Band Models

Band models are simple expressions for the mean transmission over a spectral band with many lines. Some generally available radiative transfer codes, such as MODTRAN and MDTERP, use band models.

Theoretical justification: random line spacing for random band models.
Practical justification: A fit of laboratory or line-by-line transmission data.

Regular (Elsasser) band model
Assume: evenly spaced, identical strength lines

Random band models
Assume: n randomly spaced lines in $\Delta\nu$ band ($\Delta\nu = n\delta$), lines are independent and have identical shapes, probability density of strength of i'th line is $p(S_i)$.
Different $p(S)$ give different models, e.g. Goody or Malkmus.

Random Band Models

Approach: Derive mean transmission by multiplying transmissions of each line at a particular ν, and also integrating over probability distributions of line positions ν_i and line strengths S_i for each line.

Assume lines are independent and identically distributed.

$$\bar{\mathcal{T}} = \prod_{i=1}^{n} \int_{\Delta\nu} d\nu_i \left(\frac{1}{\Delta\nu}\right) \int_0^\infty dS_i\, p(S_i) \exp\left[-uS_i f(\nu - \nu_i)\right]$$

Integral for each line in product is the same:

$$\bar{\mathcal{T}} = \left\{1 - \frac{1}{\Delta\nu} \int_{\Delta\nu} \int_0^\infty p(S) \left(1 - \exp\left[-uSf(\nu)\right]\right) d\nu dS\right\}^n$$

Define mean equivalent width by integrating over prob. dist. of S:

$$\bar{W} = \int_0^\infty p(S) \int_{\Delta\nu} \left(1 - \exp\left[-uSf(\nu)\right]\right)\, d\nu\, dS$$

Then the mean transmission is

$$\bar{\mathcal{T}} = \left\{1 - \frac{1}{n}\left(\frac{\bar{W}}{\delta}\right)\right\}^n$$

Take limit of n to infinity:

$$\bar{\mathcal{T}} = \exp\left(-\frac{\bar{W}}{\delta}\right)$$

Single line transmission is $1 - W\Delta\nu$, but for many random lines it is exponential in mean equivalent width.

Goody model has exponential line strength distribution:

$$p(S) = \frac{1}{\bar{S}} \exp(-S/\bar{S})$$

for Lorentz line shape with width $\bar{\alpha}$, integral for $\bar{W}$ gives mean transmission

$$\bar{\mathcal{T}}(u) = \exp\left[-\frac{\bar{S}u}{\delta}\left(1 + \frac{\bar{S}u}{\bar{\alpha}\pi}\right)^{-1/2}\right]$$

Malkmus model has a higher probability of weak lines:

$$p(S) \propto \frac{1}{S} \exp(-S/\bar{S})$$

for a Lorentz line shape the mean transmission is

$$\bar{\mathcal{T}}(u) = \exp\left[-\frac{\pi\bar{\alpha}}{2\delta}\left(\left[1 + \frac{4\bar{S}u}{\pi\bar{\alpha}}\right]^{1/2} - 1\right)\right]$$

The mean transmission as a function of absorber amount is in terms of two parameters, $\bar{S}/\delta$ and $\bar{\alpha}/\delta$. These parameters depend on pressure and temperature. δ is the average line spacing $\delta = \Delta\nu/n$.

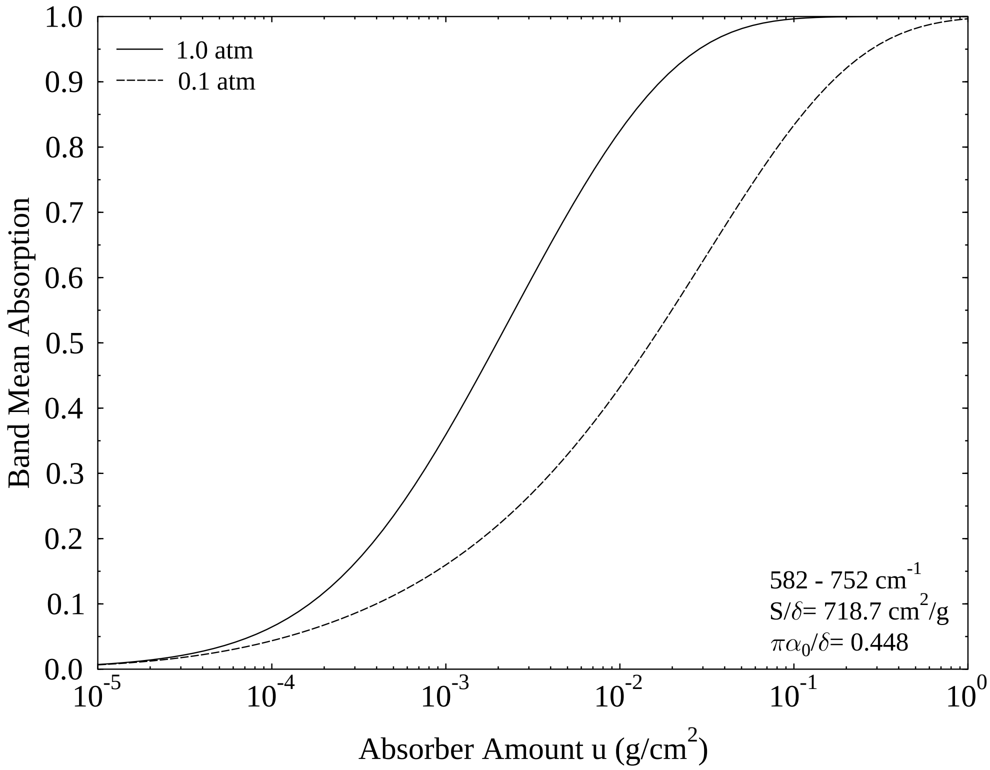
Weak line limit:

$$\frac{\bar{S}u}{\pi\bar{\alpha}} \ll 1 \quad \bar{\mathcal{T}}(u) = \exp\left[-\frac{\bar{S}u}{\delta}\right]$$

Strong line limit of Goody and Malkmus models:

$$\frac{\bar{S}u}{\pi\bar{\alpha}} \gg 1 \quad \bar{\mathcal{T}}(u) = \exp\left[-\frac{\sqrt{\pi\bar{\alpha}\bar{S}u}}{\delta}\right]$$

Goody Random Model for 15 μm CO_2 Band



Goody random band model absorption plotted as a function of absorber amount for a spectral band across the CO_2 15 μm vibrational band. The absorption is higher for the higher pressure.

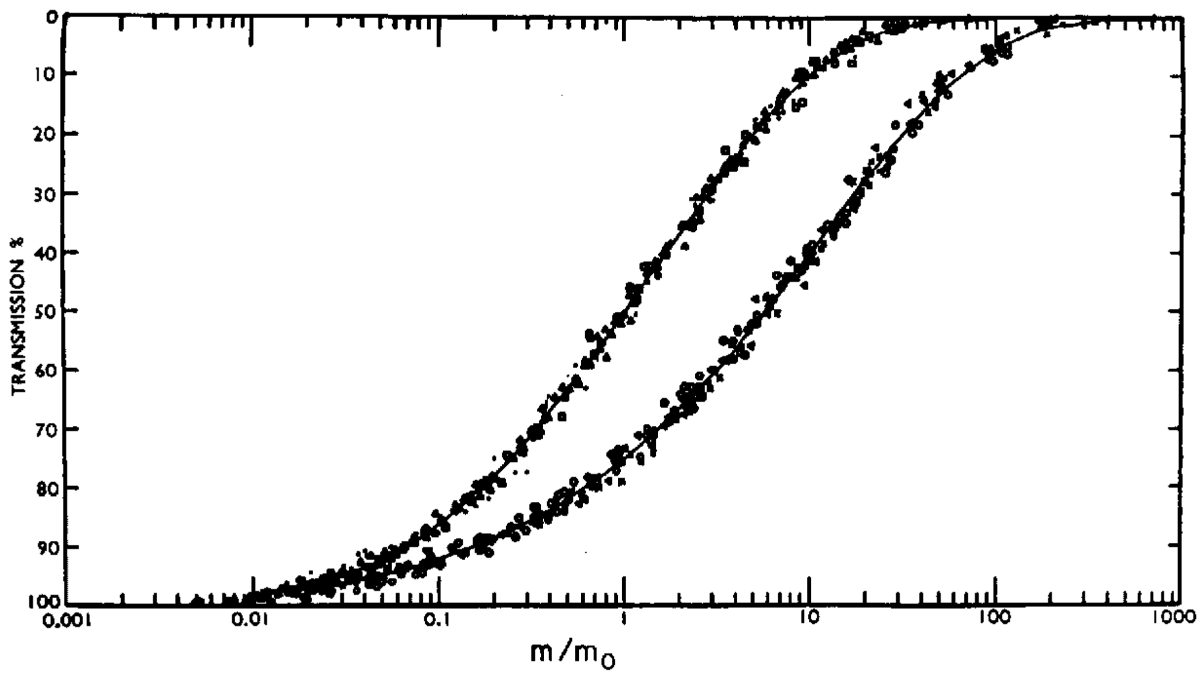
Obtaining Band Model Parameters

Band model parameters $\bar{S}$ and $\bar{\alpha}$ are derived from absorption line parameters. Usually the weak and strong limits of mean equivalent width are fit:

$$\frac{\bar{S}}{\delta} = \frac{1}{\Delta\nu}\sum_{i=1}^{n} S_i \qquad \frac{\sqrt{\pi\bar{\alpha}\bar{S}}}{\delta} = \frac{2}{\Delta\nu}\sum_{i=1}^{n}\sqrt{S_i\alpha_i} \ \ \text{(for Goody)}$$

The band model parameters are tabulated; for example, Table 17.5 in Lenoble (1993) has $\sum_{i=1}^{n} S_i$ and $\sum_{i=1}^{n}\sqrt{S_i\alpha_i}$ for 100 cm^{-1} bands at three temperatures.

How well does the Goody random band model work?



Comparison of the random band model with laboratory measurements in water vapor bands. The left curve is for a pressure of 740 Torr and the right curve is for a pressure of 125 Torr. m is the water vapor amount, and m_0 is the amount of water vapor that gives a transmission of 0.5 at 740 Torr. The symbols indicate the particular water vapor vibrational bands from 6.3 μm to 1.1 μm. [Goody & Yung, Fig. 4.18]

Inhomogeneous Paths

Band models give the transmission for a homogeneous path because band parameters are for one pressure and temperature. But band models need to be used for inhomogeneous paths to calculate the transmission between two levels.

Scaling Approximation: find an equivalent homogeneous path $\tilde{u}$ at fixed reference T_r, p_r that results in the band model having the correct transmission.

Match optical depth for line wings (centers saturated):

$$\frac{\tilde{u} S(T_r)\alpha(T_r, p_r)}{\pi(\nu - \nu_0)^2} = \int_u \frac{u S(T(u))\alpha(T(u), p(u))}{\pi(\nu - \nu_0)^2} du$$

$$\tilde{u} = \int \left(\frac{p}{p_r}\right)\left(\frac{T_r}{T}\right)^m \rho_a dz$$

Integral over height is in terms of integral over absorber amount $\int du = \int \rho_a dz$. Band model parameters computed for a reference pressure and temperature (say 500 mb, 250 K). Then the density is weighted by p and T^{-m} is integrated to get $\tilde{u}$.

van de Hulst - Curtis - Godson Approximation

More accurate band transmission for inhomogeneous paths is obtained with the *two-parameter approximation*.

Concept: adjust $\bar{S}$ and $\bar{\alpha}$ in band model to fit weak and strong line limits over inhomogeneous paths.

Strong line limit - weight line widths according to absorber amount:

$$\bar{\alpha} = \frac{\int_u \alpha(p, T)\bar{S}(T) du}{\int_u \bar{S}(T) du}$$

Often only pressure is used in scaling α:

$$\bar{\alpha} = \frac{\int_u \alpha(p) du}{u} = \frac{\alpha_0}{u}\int \frac{p}{p_0}\, \rho_a dz$$

Weak line limit - temperature effect is in adjusting line strength:

$$\bar{S} = \frac{\int_u \bar{S}(T) du}{\int_u du}$$

Curtis-Godson approximation does not work well if pressure and absorber amount are not well correlation (e.g. ozone).

Thermal Emission Radiative Transfer Revisited

Radiative Transfer Equation without scattering (using height coordinate):

$$\mu \frac{dI_\nu}{dz} = -k_\nu \rho_a \left(I - B_\nu[T(z)] \right)$$

$I_\nu(z,\mu)$ is radiance, $B_\nu[T(z)]$ is Planck function at z,
ρ_a is absorber density, and k_ν is mass absorption coefficient.
Change in radiance is difference between absorption and emission.

Integral solution for upwelling radiance assuming a black surface

$$I_\nu(z,\mu) = \mathcal{T}_\nu(z,0) I_\nu(0,\mu) + \int_0^z B_\nu[T(z')] \exp\left[-\frac{1}{\mu} \int_{z'}^z k_\nu \rho_a dz' \right] k_\nu \rho_a dz'/\mu$$

Thermal emission RTE using transmission

Can put RTE in terms of transmission between z and z' at angle μ:

$$\mathcal{T}_\nu(z,z',\mu) = \exp\left[-\frac{1}{\mu} \int_{z'}^z k_\nu \rho_a dz' \right]$$

$$\frac{d\mathcal{T}_\nu(z,z',\mu)}{dz'} = \exp\left[-\frac{1}{\mu} \int_{z'}^z k_\nu \rho_a dz' \right] \frac{k_\nu \rho_a}{\mu}$$

Upwelling radiance

$$I_\nu(z,\mu) = \mathcal{T}_\nu(z,0) B_\nu[T(0)] + \int_0^z B_\nu[T(z')] \left| \frac{d\mathcal{T}_\nu(z,z',\mu)}{dz'} \right| dz'$$

Downwelling radiance

$$I_\nu(z,\mu) = \int_z^\infty B_\nu[T(z')] \left| \frac{d\mathcal{T}_\nu(z,z',\mu)}{dz'} \right| dz'$$

Discrete solution using band model transmission

Upwelling band integrated radiance at level z_i is

$$I_{\Delta\nu}(z_i,\mu) = \mathcal{T}_{\Delta\nu}(z_i,0) B_{\Delta\nu}[T(0)] + \sum_j B_{\Delta\nu}[T_{j+1/2}] \left[\mathcal{T}_{\Delta\nu}(z_i,z_j,\mu) - \mathcal{T}_{\Delta\nu}(z_i,z_{j+1},\mu) \right]$$

where $T_{j+1/2}$ is temperature at midpoint of layer from z_j to z_{j+1}, $B_{\Delta\nu}$ is band integral of Planck function, and $\mathcal{T}_{\Delta\nu}(z_i,z_j,\mu)$ is band model mean transmission from level z_i to z_j using (scaled) absorber amount u along path at angle μ.

Weighting Functions

Weighting functions give the contribution to outgoing radiance from each level. Very important for remote sensing, but also useful for understanding IR cooling.

$$W_\nu(z,\infty) = \left|\frac{d\mathcal{T}_\nu(z,\infty)}{dz}\right|$$

Upwelling radiance is then an integral over the weighting function

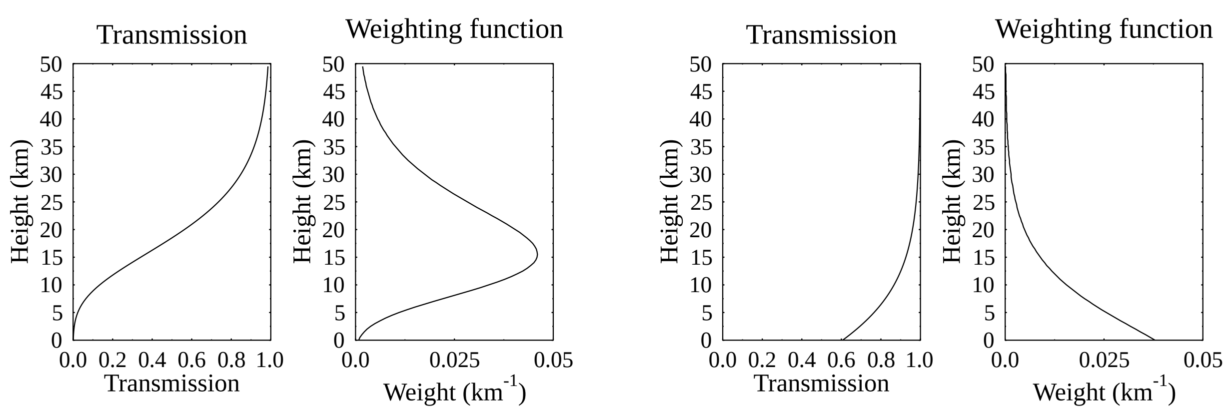
$$I_\nu(\infty,\mu) = \mathcal{T}_\nu(\infty,0)I_\nu(0,\mu) + \int_0^\infty B_\nu[T(z)]\, W_\nu(z,\infty)dz$$

Weighting function referenced to surface for contribution to downwelling radiance at surface

$$W_\nu(0,z) = \left|\frac{d\mathcal{T}_\nu(0,z)}{dz}\right|$$

What do weighting functions look like?

Schematic for optically thick and optically thin cases from space:

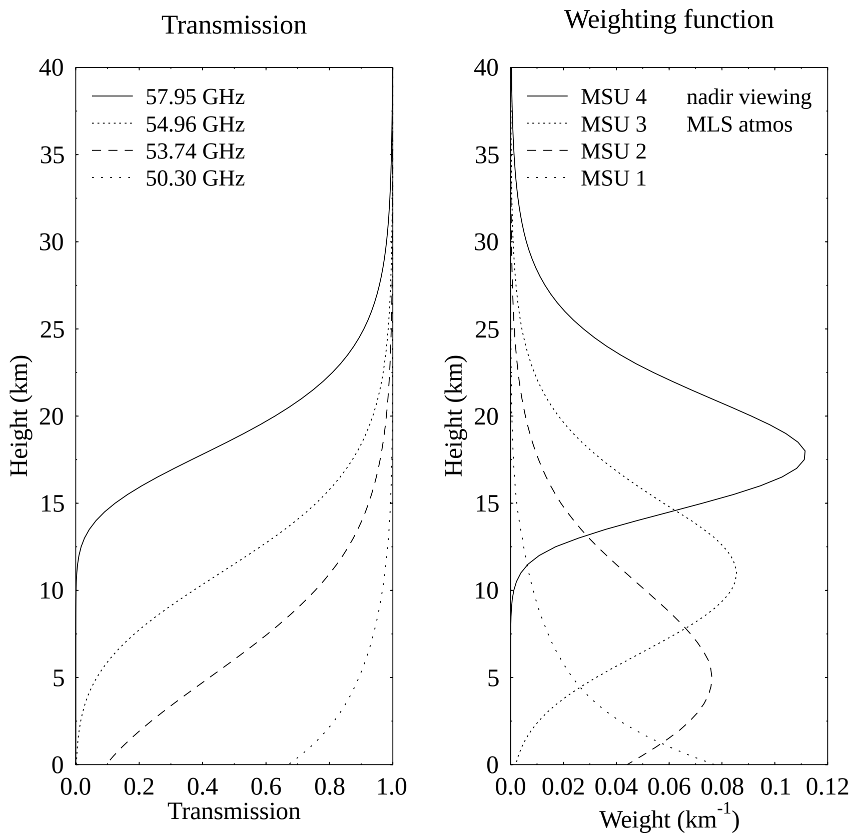


Why do weighting functions look like this?

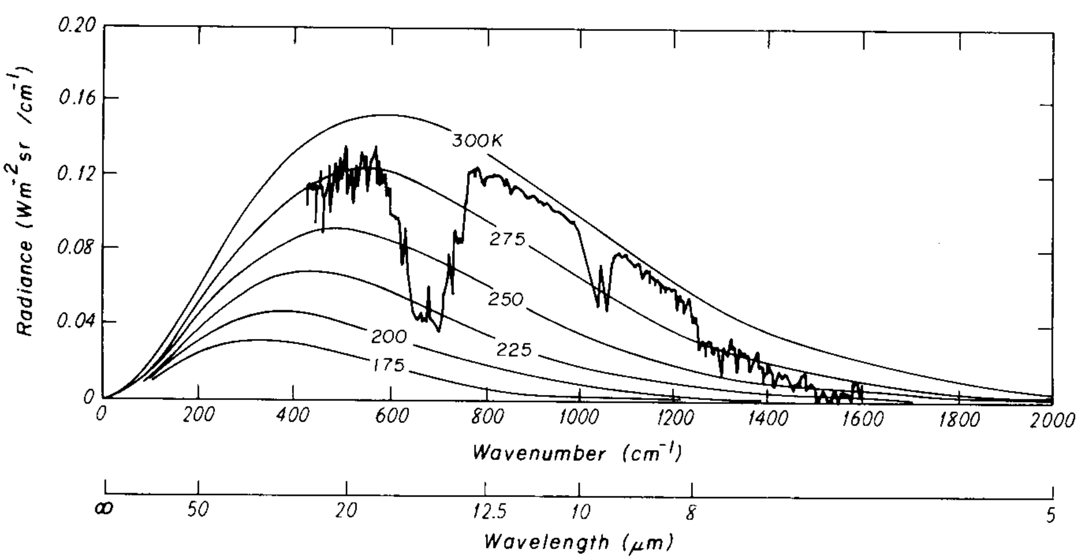
Two factors in weighting function - transmission and extinction

$$W_\nu(z,\infty) \;=\; e^{-\tau(z)/\mu}\,\frac{k_\nu \rho_a}{\mu}$$

Transmission decreases away from observer, extinction decreases upward.



Transmission profiles and weighting functions referenced to space for the Microwave Sounding Unit on the NOAA polar orbiting satellites.



An upwelling Earth radiance spectrum measured by the Infrared Interferometer Spectrometer aboard the Nimbus 4 satellite. Planck radiance curves are also shown. [Liou, 1992; Fig 2.1.]

Heating Rates

Calculating broadband longwave fluxes requires three integrals:
1) over height in RTE, 2) over angle to get flux, 3) over spectrum.

Net flux - net power per area passing through level

$$F_{net} = F\uparrow - F\downarrow = 2\pi \int_{-1}^{1} I(\mu)\mu d\mu$$

Heating (or cooling) from broadband net flux convergence:

$$\left.\frac{dT}{dt}\right|_{rad} = -\frac{1}{\rho C_p}\frac{dF_{net}}{dz} = \frac{g}{C_p}\frac{dF_{net}}{dp}$$

Heating from absorption, cooling from emission.

Consider a layer:
flux entering $= F^{\uparrow}_{bot} + F^{\downarrow}_{top}$
flux leaving $= F^{\downarrow}_{bot} + F^{\uparrow}_{top}$
Net flux convergence = absorbed - emitted = entering - leaving

Example: Longwave radiative cooling at night.
US standard atmosphere, 0 to 1 km layer (fluxes calculated with MODTRAN).
$F^{\downarrow}_1 = 250\ \mathrm{W/m^2}$ $F^{\uparrow}_0 = 390\ \mathrm{W/m^2}$ entering = 640
$F^{\downarrow}_0 = 285\ \mathrm{W/m^2}$ $F^{\uparrow}_1 = 375\ \mathrm{W/m^2}$ leaving = 660
Net flux converged = $-20\ \mathrm{W/m^2}$

$$dT/dt = -\frac{1}{\rho C_p}\frac{\Delta F_{net}}{\Delta z} = \frac{-20\ \mathrm{J\ s^{-1} m^{-2}}}{(1.17\ \mathrm{kg/m^3})(1004\ \mathrm{J\ kg^{-1} K^{-1}})(1000\ \mathrm{m})}$$

$$dT/dt = -1.7 \times 10^{-5}\ \mathrm{K/s} = -1.5\ \mathrm{K/day}$$

Net flux convergence is usually computed by differencing discrete layer fluxes. May get numerical errors from differencing almost equal up and down fluxes in opaque atmospheres. Curtis matrix approach used for Venus.

Exchange of Flux between Layers

Derive net flux convergence from radiative transfer equation to understand sources of heating/cooling at a level.
The spectral net flux convergence may be obtained from RTE solution by taking d/dz of $F^{\uparrow} - F^{\downarrow}$:

$$\begin{aligned} \frac{dF_{\nu,net}}{dz} &= \pi B_\nu(0)\frac{\partial \mathcal{T}_\nu^f(0,z)}{\partial z} + \int_0^z \pi B_\nu(z')\frac{\partial^2 \mathcal{T}_\nu^f(z',z)}{\partial z \partial z'}dz' \\ &+ \int_z^\infty \pi B_\nu(z')\frac{\partial^2 \mathcal{T}_\nu^f(z,z')}{\partial z \partial z'}dz' \end{aligned}$$

$\mathcal{T}_\nu^f(z,z')$ is *flux* transmission between levels z and z'.
Add and subtract in two terms like

$$\pi B_\nu(z)\int_z^\infty \frac{\partial^2 \mathcal{T}_\nu^f(z,z')}{\partial z \partial z'}dz' = \pi B_\nu(z)\frac{\partial \mathcal{T}_\nu^f(z,\infty)}{\partial z}$$

Final result for exchange of flux between layers form of net flux divergence:

$$\begin{aligned} \frac{dF_{\nu,net}}{dz}(z) &= \pi B_\nu(z)\frac{\partial \mathcal{T}_\nu^f(z,\infty)}{\partial z} \\ &+ \pi[B_\nu(0) - B_\nu(z)]\frac{\partial \mathcal{T}_\nu^f(0,z)}{\partial z} \\ &+ \int_0^z \pi[B_\nu(z') - B_\nu(z)]\frac{\partial^2 \mathcal{T}_\nu^f(z',z)}{\partial z \partial z'}dz' \\ &+ \int_z^\infty \pi[B_\nu(z') - B_\nu(z)]\frac{\partial^2 \mathcal{T}_\nu^f(z,z')}{\partial z \partial z'}dz' \end{aligned}$$

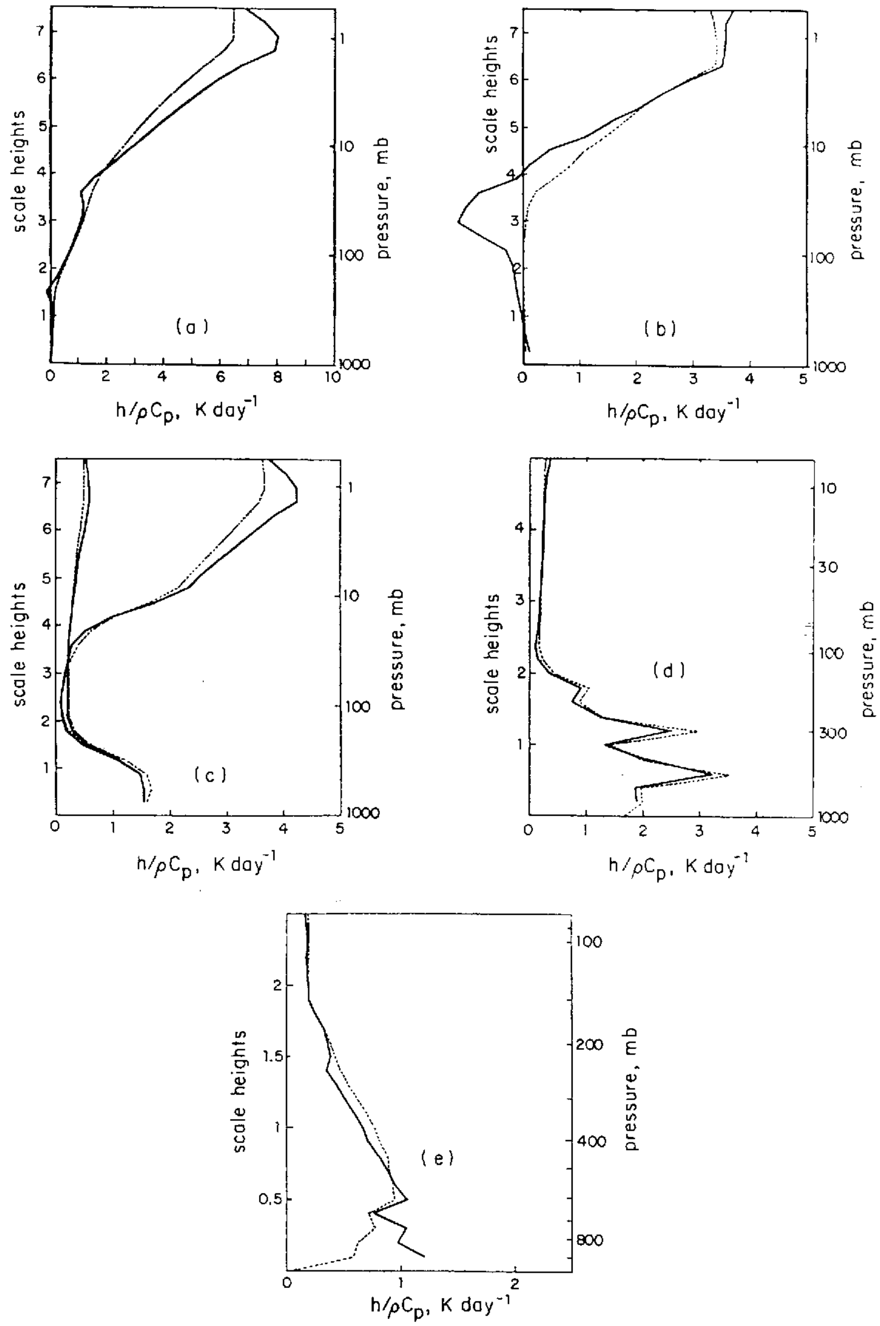
First term is **cooling to space** - often a good approximation.
Cooling to space is simply the weighting function times the Planck flux.
Second term is exchange of energy with surface.
Important if large temperature difference and transmission is high.
Integrals are exchange between levels below and above.

Role of second derivative of transmission can be understood using

$$\frac{\partial^2 \mathcal{T}_\nu^f(z',z)}{\partial z \partial z'} \sim -\beta(z')\beta(z)\mathcal{T}_\nu^f(z',z)$$

β is emissivity/absorptivity of thin layer.

See how well cooling to space works:



Cooling to space compared to total cooling. The solid lines include all terms in the heating equation; the dotted line is calculated from the cooling to space approximation. The vertical scale is $-\ln(p(z)/p_{sfc})$. (a) CO_2 15 μm band for a midlatitude temperature profile. (b) O_3 9.6 μm band for a tropical temperature profile. (c)H_2O for a tropical temperature profile with wet and dry stratospheres. (d) H_2O for a tropical temperature profile. (e) H_2O for a mid-winter, arctic temperature profile. [from Rodgers and Walshaw (1966)] [Goody & Yung, Fig. 6.14]

Alternative Method for Heating Rates

Can derive net flux convergence by integrating RTE over $d\mu d\phi$

$$\int_0^{2\pi} \int_{-1}^{1} \left\{ \mu \frac{dI}{dz} = -\beta_{abs} \left[I(\mu) - B \right] \right\} d\mu d\phi$$

$$\frac{dF_{net}}{dz} = -4\pi\beta_{abs}(\bar{I} - B)$$

where $\bar{I}$ is the mean radiance and β_{abs} is volume absorption coefficient.

Divergence of flux - cooling from Planck emission,
Convergence of flux - heating from absorption of radiation.

MDTERP

MDTERP (Maryland Terrestial Radiation Package) is a narrow-band longwave radiative transfer model with a graphical user interface developed by Robert Ellingson and Ezra Takara at the University of Maryland.

MDTERP - Description

Quantities Calculated

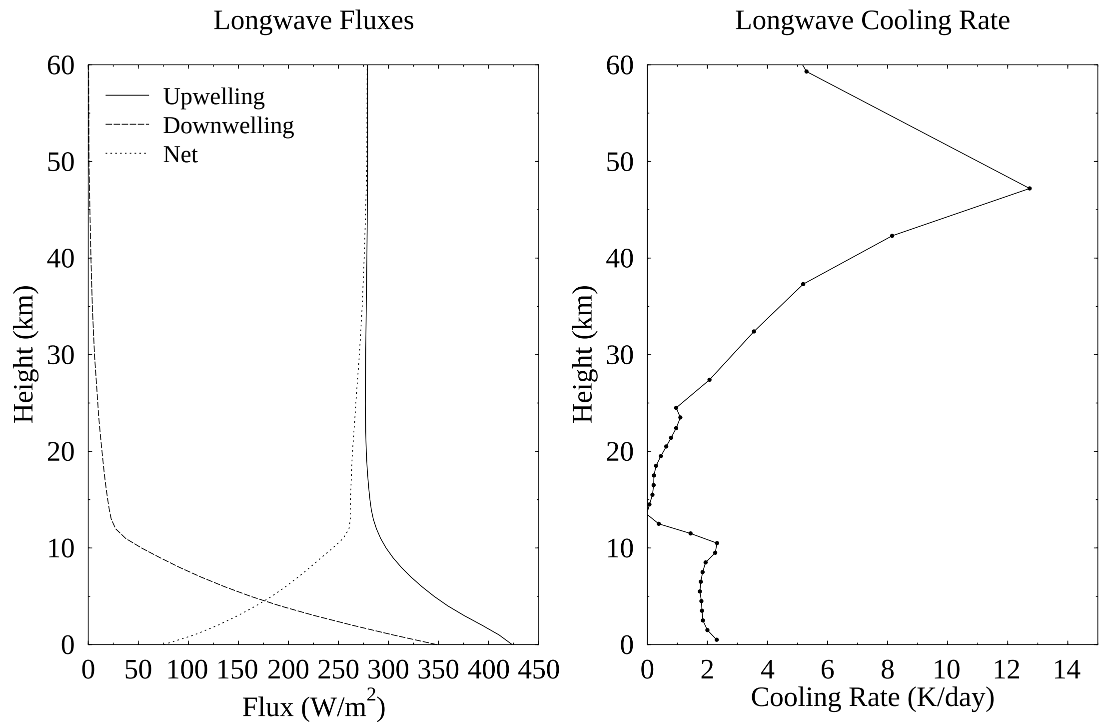
- Spectral radiance as F(wavenumber, nadir angle and altitude)
- Upward, downward and net upward fluxes and heating rates as F(wavenumber and altitude)

Assumptions/Features

- Plane parallel, LTE, black surface and five angles in each hemisphere
- Molecular absorption by H2O, CO2, O3, CH4, N2O (1992 HITRAN)
- 10 cm^{-1} resolution from 0 to 3000 cm^{-1}
- Transmittances for for all gases fitted to LBLRTM (H2O - empirical model; others - Malkmus model)
- Fluxes and heating rates calibrated with LBLRTM
- As many as six cloud layers (black in current version)
- User control of input data, calculation levels, & cloud positions
- User selectable graphical and/or text output

Broadband Fluxes and Heating Rates from MDTERP

MDTERP profiles for midlatitude summer atmosphere



MDTERP profiles for midlatitude winter atmosphere