Radiative and Radiative-Convective Equilibrium Models

Topics:
1. Eddington gray radiative equilibrium model
2. Radiative equilibrium solution methods
3. Radiative-convective equilibrium

Reading: Liou 8; Thomas 12.1-12.7

Radiative equilibrium models predict the atmospheric temperature profile of an atmosphere in radiative equilibrium ($\frac{dF_{net}}{dz} = 0$). If we assume the atmosphere is spectrally gray, then we can solve for the temperature profile analytically.

Eddington Gray Radiative Equilibrium Model

Assume: 1) Radiative equilibrium: $\frac{dF_{net}}{d\tau} = 0$.
2) Gray (spectrally uniform) nonscattering atmosphere,
3) Eddington approximation: $I(\mu) = I_0 + I_1 \mu$
4) Solar flux absorbed only at surface.

Eddington approximation $\rightarrow$ $F_{net} = \frac{4\pi}{3} I_1$

Integrate RTE over $d\mu$: $2\pi \int_{-1}^{1} \left[\mu \frac{dI}{d\tau} = I - B \right] d\mu$

$$\frac{dF_{net}}{d\tau} = 4\pi I_0 - 4\pi B \quad \rightarrow \quad I_0 = B$$

Integrate RTE over $\mu d\mu$: $\frac{4\pi}{3} \frac{dI_0}{d\tau} = F_{net}$

$$B(\tau) = B(0) + \frac{3}{4\pi} F_{net}\, \tau$$

Constants $B(0)$ and F_{net} determined from boundary conditions.
Top: 1) zero incident longwave flux,
2) outgoing longwave flux equals absorbed solar flux $F_{sun} = \frac{S}{4}(1 - \bar{r})$.
Bottom: downwelling SW + LW flux equals surface emission.

Eddington Gray Radiative Equilibrium Results

Longwave flux profile:

$$F^{\pm} = \pi B \pm \frac{1}{2} F_{net} \quad F^{\uparrow} = F_{sun}\left(1 + \frac{3}{4}\tau\right) \quad F^{\downarrow} = F_{sun}\left(\frac{3}{4}\tau\right)$$

Atmosphere blackbody emission and temperature profiles:

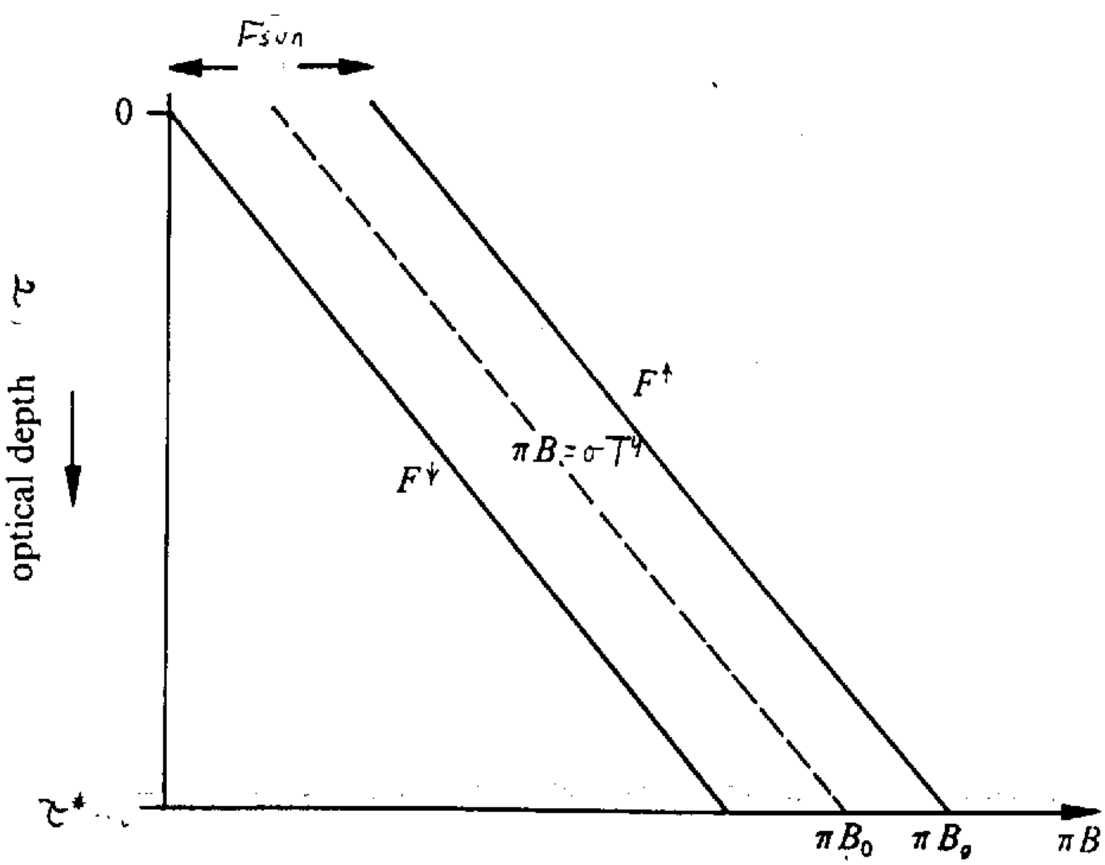
$$\pi B(\tau) = \frac{F_{sun}}{2}\left(1 + \frac{3}{2}\tau\right) \qquad T^4(\tau) = T_e^4\left(\frac{1}{2} + \frac{3}{4}\tau\right)$$

Surface temperature is discontinuous with atmosphere (hotter):

$$\pi B_s = \pi B(\tau^*) + \frac{F_{sun}}{2} \qquad T_s^4 = T_e^4\left(1 + \frac{3}{4}\tau^*\right)$$

Greenhouse effect - larger τ^* increases surface temperature.

Runaway greenhouse effect (Venus): as $\tau^* \to \infty$, $T_s \to \infty$
Higher temperature, more greenhouse gases (positive feedback).



Profiles of upwelling, downwelling, and emitted flux for gray radiative equilibrium. [Houghton, 1986]

Eddington Gray Radiative Equilibrium Temperatures

If we want temperature profile in terms of height, we need to relate optical depth to height.

Assume exponential profile of absorber $\rho_a(z) = \rho_0 \exp(-z/H_a)$

Optical depth profile is

$$\tau(z) = \bar{k_a} \int_z^\infty \rho_a(z) dz \; = \bar{k_a} \rho_0 H_a \exp(-z/H_a) \; = \tau^* \exp(-z/H_a)$$

Temperature profile:

$$T(z) = T_e \left[\frac{1}{2} + \frac{3}{4} \tau^* \exp(-z/H_a) \right]^{1/4}$$

Lapse rate:

$$\frac{dT}{dz}(z) = -\frac{3}{8} \frac{\tau^*}{1 + \frac{3}{2}\tau^*} \frac{T(z)}{H_a} \exp(-z/H_a)$$

Low optical depth → stable stratosphere.
Smaller scale height of absorber causes steeper lapse rate.
Steepest lapse rate near surface (z=0).

Derivation of the Eddington Gray Radiative Equilibrium

Assumptions:

1. Radiative equilibrium: $\frac{dF_{net}}{d\tau} = 0$
2. Gray atmosphere in longwave
3. No scattering and black surface in longwave
4. No solar absorption in atmosphere
5. Eddington approximation: $I(\mu) = I_0 + I_1\mu$

Since the atmosphere is gray (all wavelengths are equivalent) we can start with the wavelength integrated thermal emission radiative transfer equation for an atmosphere with no scattering:

$$\mu \frac{dI}{d\tau} = I - B$$

where I is the integrated radiance (W m^{-2} st^{-1}), the optical depth τ increases downward, and $\mu > 0$ is the upward direction. The isotropic blackbody emission B is the integrated Planck function. Since the blackbody emission is a function only of temperature ($\pi B(T) = \sigma T^4$), deriving the variation of B with optical depth is equivalent to determining the temperature structure of the atmosphere.

Using the Eddington approximation the net flux (positive upward) becomes

$$F_{net} = 2\pi \int_{-1}^{1} I\mu \, d\mu = \frac{4\pi}{3} I_1$$

The radiative equilibrium assumption implies that F_{net} (and I_1) is constant with optical depth. Integrating the RTE over $d\mu$ gives

$$2\pi \frac{d}{d\tau} \int_{-1}^{1} I\mu \, d\mu = 2\pi \int_{-1}^{1} I \, d\mu - 2\pi \int_{-1}^{1} B \, d\mu$$

$$\frac{dF_{net}}{d\tau} = 4\pi I_0 - 4\pi B \ .$$

The assumption of radiative equilibrium then implies that

$$I_0 = B \ .$$

Integrating the RTE over $\mu \, d\mu$ gives

$$2\pi \frac{d}{d\tau} \int_{-1}^{1} I\mu^2 \, d\mu = 2\pi \int_{-1}^{1} I\mu \, d\mu - 2\pi \int_{-1}^{1} B\mu \, d\mu \ .$$

Since B is isotropic the last term drops out leaving

$$\frac{4\pi}{3} \frac{dI_0}{d\tau} = F_{net} = \frac{4\pi}{3} I_1 \ .$$

$$\frac{dB}{d\tau} = I_1 \ .$$

Thus the solution for B is simply a linear function of optical depth:

$$B(\tau) = B(0) + I_1 \, \tau \ \ .$$

The constants in this linear relationship ($B(0)$ and I_1) are determined from the boundary conditions. The first boundary condition at the top of the atmosphere is that there is no downwelling longwave flux:

$$F^{\downarrow}(0) = 2\pi \int_{-1}^{0} I(\mu)\mu d\mu \ = \pi B(0) - \frac{2\pi}{3} I_1 \ = 0$$

From this we have $I_1 = \frac{3}{2} B(0)$ or $F_{net} = 2\pi B(0)$. The second boundary condition at the top of the atmosphere is that the upwelling longwave flux is equal to the absorbed solar flux F_{sun}:

$$F^{\uparrow}(0) = 2\pi \int_{0}^{+1} I(\mu)\mu d\mu \ = [\pi B(0) + \frac{2\pi}{3} I_1] \ \ = F_{sun}$$

Putting in $I_1 = \frac{3}{2} B(0)$ gives

$$F_{sun} \ = \ 2\pi B(0) \ = \ F_{net}$$

So now we have $B(0)$ and I_1 and thus the atmosphere Planck function profile is determined:

$$B(\tau) = \frac{F_{sun}}{2\pi} \left(1 + \frac{3}{2}\tau\right) \ \ .$$

Furthermore, the net longwave flux upwelling throughout the atmosphere equals the absorbed shortwave flux.
The final step is to apply the boundary condition at the surface to obtain the surface temperature. This boundary condition is that the emitted flux by the surface equals the sum of the downwelling shortwave and longwave flux at the black surface:

$$F_{sun} + F^{\downarrow}(\tau^*) \ = \ \pi B_s \qquad F^{\downarrow}(\tau^*) = \pi B(\tau^*) - \frac{2\pi}{3} I_1$$

Using $F_{sun} = \frac{4\pi}{3} I_1$ gives the emission from the surface

$$B_s = B(\tau^*) + \frac{F_{sun}}{2\pi} \ ,$$

which is discontinuous with the atmospheric emission.
The previous results may be expressed in terms of temperature by

$$\begin{aligned} T^4(\tau) &= T_e^4 \left(\frac{1}{2} + \frac{3}{4}\tau\right) \\ T_t^4 &= \frac{1}{2} T_e^4 \\ T_s^4 &= T_e^4 \left(1 + \frac{3}{4}\tau^*\right) \ \ . \end{aligned}$$

The definition of planetary equilibrium temperature is $\sigma T_e^4 = F_{sun} = \frac{S}{4}(1 - \bar{r})$, where S is the solar constant , and $\bar{r}$ is the planetary albedo. For the earth $S = 1366$ W/m^2 and $\bar{r} = 0.30$ implies a planetary equilibrium temperature of $T_e = 255$ K and a “top” temperature of $T_t = 214$ K. Assuming a global averaged surface *air* temperature of $T(\tau^*) = 288$ K gives a gray body optical depth of $\tau^* = 1.5$, and a surface skin temperature of $T_s = 308$ K.

Radiative Convective Equilibrium Models

- These climate models solve for the vertical profile of temperature using accurate broadband radiative transfer models.
- Radiative equilibrium means zero heating rate, $\frac{dF_{net}}{dz} = 0$.
- Model inputs vertical profile of gases, aerosols, and clouds.
  Iterates temperature profile to achieve equilibrium.
- Climate feedbacks can be included by having water vapor,
  surface albedo, clouds, etc. depend on temperature.

Solving for Radiative Equilibrium

Iterate temperature profile $T(z)$ to get zero radiative heating rate $\frac{\partial T(z)}{\partial t} = 0$.

1. *Time marching method:*
   Temperature at $t+1$ time step from heating rate at time t.

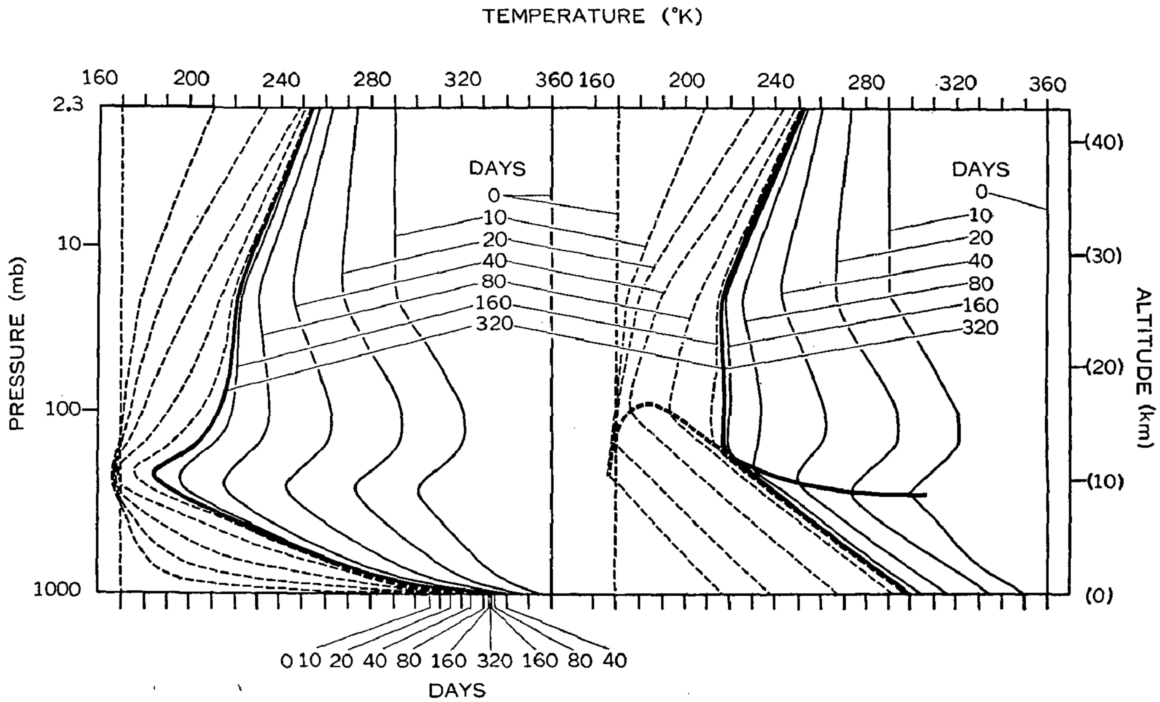
$$T^{t+1}(z_k) = T^t(z_k) + \left(\frac{\partial T(z_k)}{\partial t} \right)^t \Delta t$$

2. Direct solver: use gradient information in nonlinear root solver.
   Faster, but more complex than time marching.

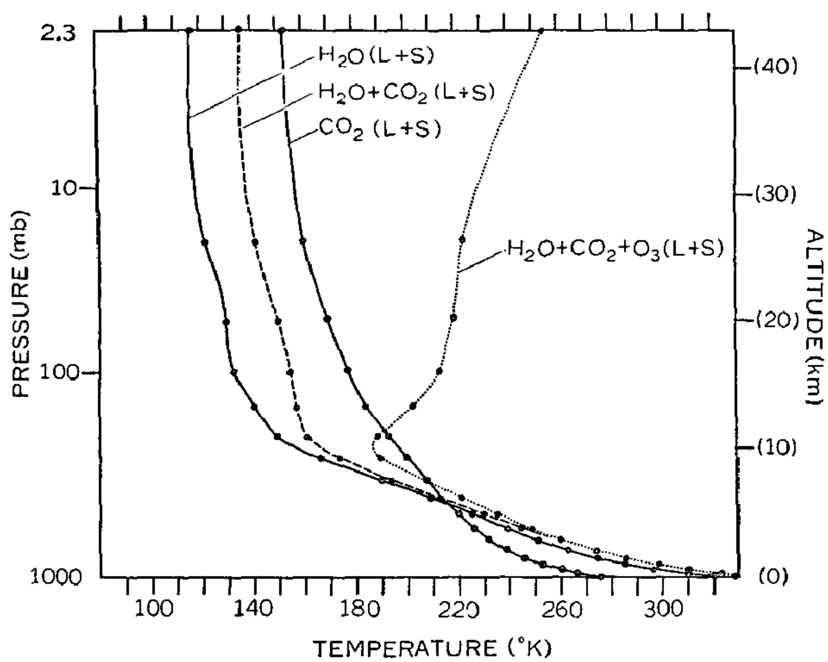
Time marching convergence models physical radiative relaxation.

Results: The radiative equilibrium surface temperature is too high and the temperature profile is unrealistic.

Radiative equilibrium temperature profiles show:
  CO_2 only atmosphere has less steep profile.
  Earth's stratosphere due to UV absorption by ozone.
  Most greenhouse effect from water vapor.



The approach to pure radiative equilibrium (left) and radiative-convective equilibrium (right) using the time marching method starting with hot and cold isothermal atmospheres. [Manabe and Strickler, 1964]



Pure radiative equilibrium temperature profiles for various atmospheric molecular absorbers in a clear sky at 35 N in April. L+S means that the effects of both longwave and shortwave radiation are included. [Manabe and Strickler, 1964]

Radiative-Convective Equilibrium

Problem: radiative equilibrium temperature lapse rate near surface exceeds threshold for convection.

Fix: assume convection limits lapse rates to $\leq \Gamma_c$ (e.g. 6.5 K/km).

Radiative-convective equilibrium is equilibrium of radiative + convective fluxes.

Convective Adjustment methods:

1. Move heat like convection: If Γ_c exceeded, adjust temperatures so Γ_c achieved and heat is conserved.
   Iterate - mix layers starting from bottom until no instability.

2. Parameterize convective flux, e.g.

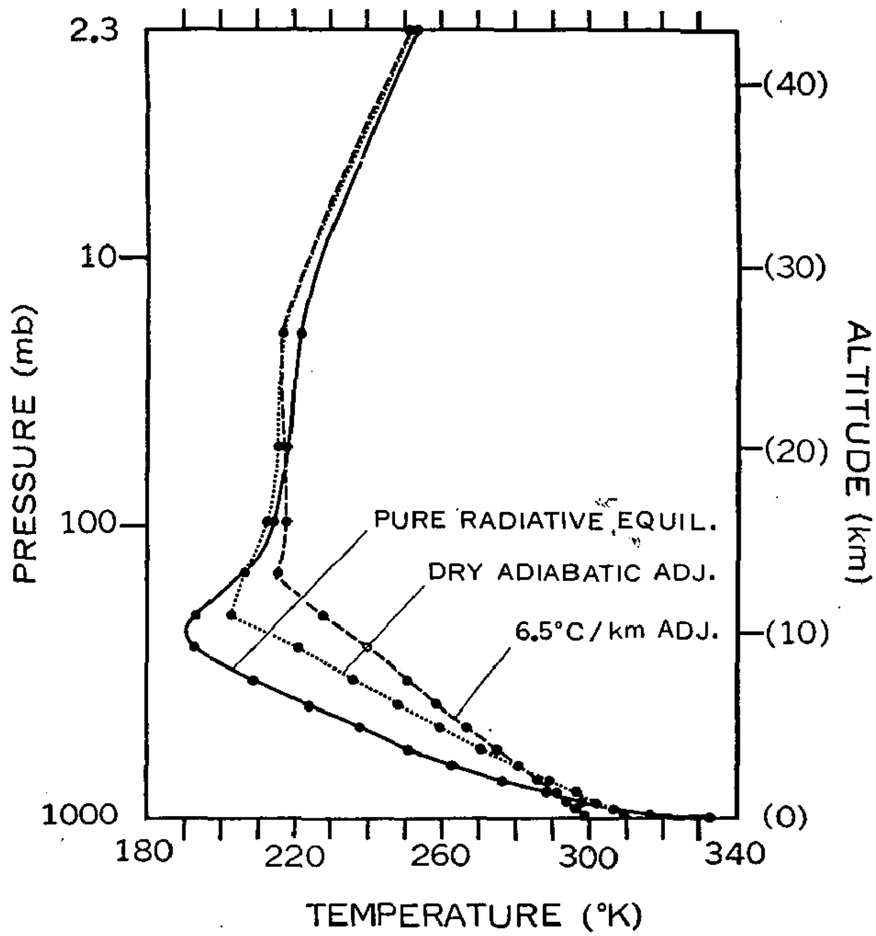
$$F_{conv} = C \left(\left| \frac{dT}{dz} \right| - \Gamma_c \right) \quad \text{if} \quad \left| \frac{dT}{dz} \right| > \Gamma_c$$

   $\frac{\partial F_{net}}{\partial dz} = 0$ applies to the sum of radiative and convective net flux.

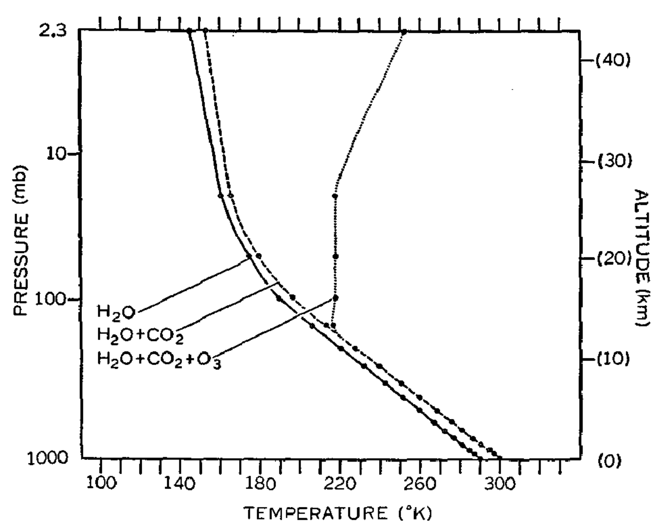
RCE Model Results

Manabe and Strickler (1964) used fixed absolute humidity.

Radiative equilibrium is fairly accurate for stratosphere (though latitudinal and seasonal dependence is not correct).
Convection required for reasonable tropospheric temperatures.



Pure radiative equilibrium and radiative-convective equilibrium temperature profiles for two values of Γ_c for clear sky. [Manabe and Strickler, 1964]



Radiative-convective equilibrium profiles for various atmospheric molecular absorbers in a clear sky at 35 N in April. [Manabe and Strickler, 1964]

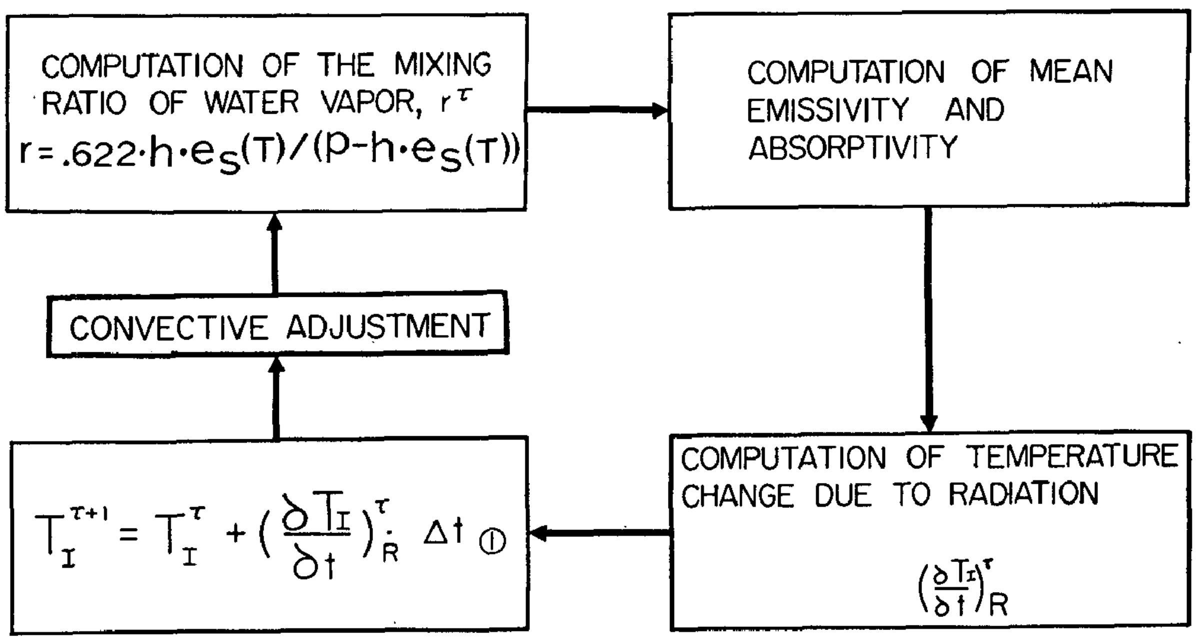
Water Vapor Feedback with RCE Model

Manabe and Wetherald (1967) assumed fixed profile of relative humidity (linearly decreasing with height in troposphere). Actual water vapor feedback is more complicated (dynamics, clouds, precipitation, etc.).

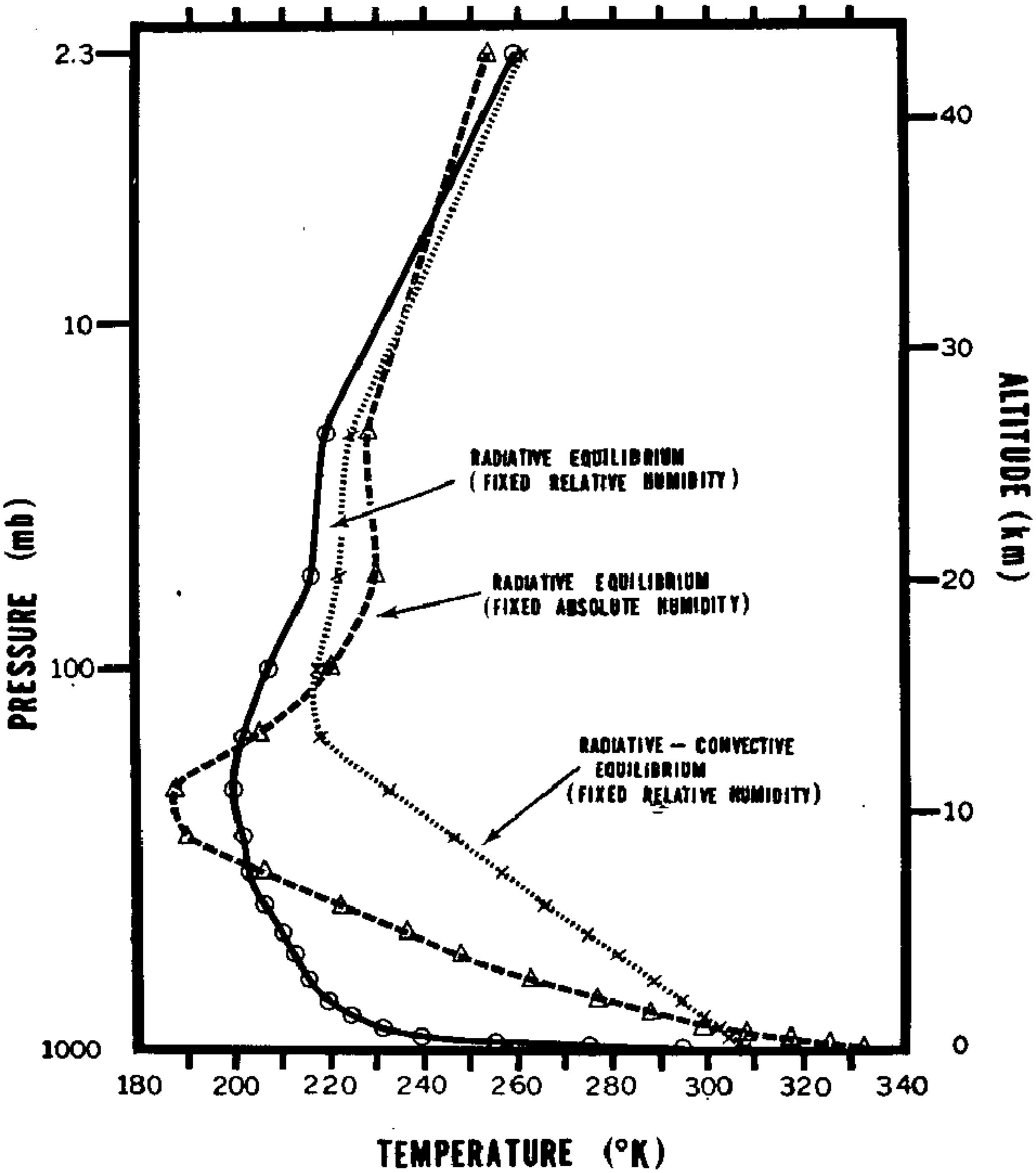
Warming → more water vapor → higher atmos emissivity
→ more greenhouse effect → more temperature increase.

Gives much steeper radiative equilibrium lapse rate than fixed absolute humidity.

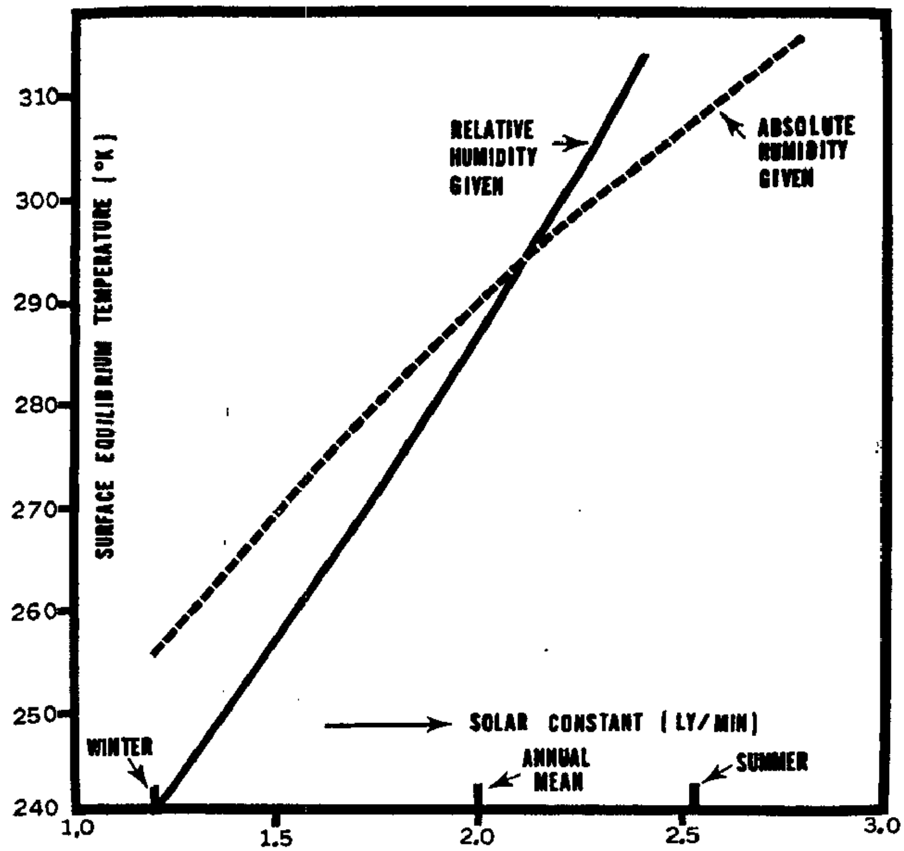
RCE shows increased climate sensitivity from water vapor feedback:
larger surface temperature change for same radiative forcing.



Flowchart for the time-marching numerical integration. [Manabe and Wetherald, 1967]



Temperature profiles for pure radiative equilibrium with fixed absolute humidity, pure radiative equilibrium with fixed relative humidity, and radiative-convective equilibrium with fixed relative humidity (all clear sky). [Manabe and Wetherald, 1967]



Climate sensitivity in terms of surface temperature versus solar constant for fixed absolute humidity (no climate feedbacks) and for fixed relative humidity (water vapor feedback). [Manabe and Wetherald, 1967]

TABLE 4. Equilibrium temperature of the earth's surface (°K) and the CO_2 content of the atmosphere.

CO_2 content (ppm)	Average cloudiness		Clear	
	Fixed absolute humidity	Fixed relative humidity	Fixed absolute humidity	Fixed relative humidity
150	289.80	286.11	298.75	304.40
300	291.05	288.39	300.05	307.20
600	292.38	290.75	301.41	310.12

TABLE 5. Change of equilibrium temperature of the earth's surface corresponding to various changes of CO_2 content of the atmosphere.

Change of CO_2 content (ppm)	Fixed absolute humidity		Fixed relative humidity	
	Average cloudiness	Clear	Average cloudiness	Clear
300 → 150	−1.25	−1.30	−2.28	−2.80
300 → 600	+1.33	+1.36	+2.36	2.92