Week 1: August 27-31

Radiative Quantities and Emission

Topics:
1. Introduction
2. Nature of radiation
3. Definition of radiative quantities
4. Emission of radiation
5. Planck function and laws

Reading: Liou section 1.1-1.2; Thomas & Stamnes 2.1-2.6; 4.3.1-4.3.2

Introduction to Radiative Processes

- What is "radiative processes"?
  How electromagnetic radiation interacts with the surface and the gases and particles in an atmosphere.

- **Radiative Transfer:** given surface and atmospheric properties calculate the radiation field in the atmosphere.

  1. **Molecular absorption:** attenuation of radiation by gases.
  2. **Particle scattering:** absorption and redirection of radiation by particles and density fluctuations in a fluid.
  3. **Radiative transfer:** calculation of the radiation field from *optical properties* (from 1 and 2) and boundary conditions.

Radiative Transfer Applications

- What is it good for?
  RT is a key part of *remote sensing* and *climate modeling*.

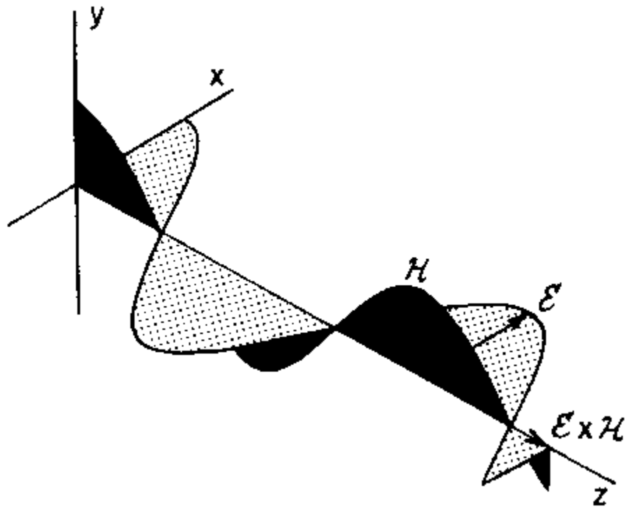
- Remote sensing: retrieving atmosphere or surface properties from measured radiances. Inversion of radiative transfer.

- Climate: radiative heating rates affect temperatures, hence atmosphere dynamics, which affect the atmospheric state.

What is Radiation?

- Classically, radiation is oscillating electric and magnetic fields.
  E field perpendicular to propagation direction.
  E field is a two component vector: the relative amplitudes and phases of the E field components give rise to polarization.
  Intensity (or energy) is proportional to square of electric field.

- Radiation properties: Intensity, Phase, Polarization.
- Radiation depends on: Frequency, Space, Time, Direction.



A schematic view of an electromagnetic wave propagating along the z axis. From Maxwell's equations the E and M fields are 90° out of phase and perpendicular to each other and the direction of travel.

The Spectrum

- **Wavelength** λ related to frequency $\tilde{\nu}$ and speed of light c by

$$\lambda = \frac{c}{\tilde{\nu}} \qquad c = 2.998 \times 10^8 \text{ m/s}$$

- **Wavenumber** ν is number of waves in a given length (usually 1 cm) and is proportional to frequency:

$$\nu = \frac{1}{\lambda} \;=\; \frac{10000 \text{ cm}^{-1}\,\mu\text{m}}{\lambda}$$

  Example: 8-12 μm atmospheric window is 833-1250 cm^{-1}.

We will mainly use wavelength and wavenumber. Wavenumber is used especially for molecular absorption spectroscopy.

Wavelengths (μm)	Spectral region
0.1 - 0.38	Ultraviolet
0.38 - 0.75	Visible
0.75 - 4	Near infrared
4 - 20	mid infrared
20 - 1000	far infrared
$>$1000	microwave
0.1 - ≈4.0	Shortwave (from sun = solar)
≈4.0 - 1000	Longwave (from earth = terrestrial)

Particulate nature of radiation

- Quantum mechanics says that radiation behaves as particles, called photons.
- The energy of a photon is

$$E = h\tilde{\nu} = hc/\lambda$$

  where h is Planck's constant ($h = 6.626 \times 10^{-34}$ J s).

- The quantized nature of light is most important when considering absorption of radiation by atoms and molecules.

Solid angle

Direction of radiation: θ is polar or zenith angle, ϕ is azimuth angle.
Solid angle = area intercepted on sphere of unit radius

$$\Omega = \int d\Omega = \int \int \sin\theta d\theta d\phi$$

Units of solid angle are steradian (sr).

Change of variables: use (μ,ϕ) for directions, $\mu = \cos\theta \quad d\Omega = d\mu d\phi$
Solid angle for all directions: $\Omega = \int_0^{2\pi} \int_{-1}^{+1} d\mu d\phi \ = 4\pi$
$\mu = 1$ ($\theta = 0$) is towards zenith, $\mu = -1$ ($\theta = 180$) is towards nadir,
$\mu = 0$ ($\theta = 90$) is towards horizon.
Warning: sometimes we will keep $\mu > 0$ even for downwelling.

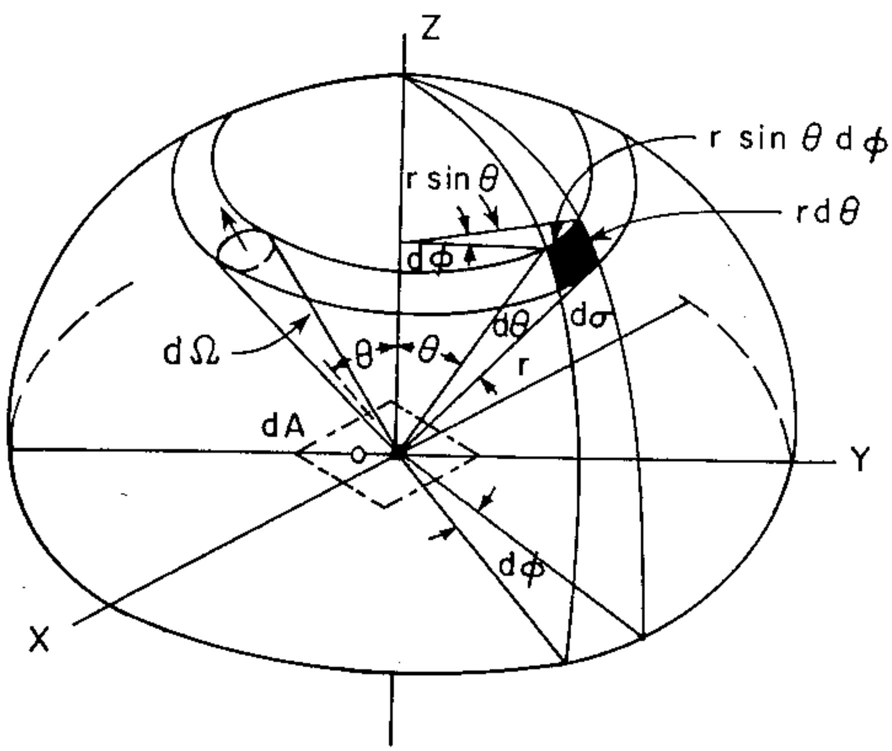


Illustration of differential solid angle $d\Omega$ in spherical coordinates for cone of radiation around zenith angle θ and azimuth angle ϕ.

Example: What is the solid angle of the Sun from the Earth? The Sun is $r_\oplus = 1.50 \times 10^8$ km from the Earth and has a radius of $R_\odot = 6.96 \times 10^5$ km.

$$\Omega = \frac{\pi R_\odot^2}{r_\oplus^2} = 6.76 \times 10^{-5} \text{ sr}$$

Radiative Quantities

Radiative Definitions

- Monochromatic *intensity* or *radiance* is the basic measure of radiation
  = energy per area per solid angle per spectral interval per time.

$$I_\lambda = \frac{dE}{\cos\theta\; dA\; d\Omega\; d\lambda\; dt} \qquad \mathrm{Units:}\;\; \mathrm{W\ m^{-2}\ ster^{-1}\ \mu m^{-1}}$$

- Monochromatic *flux* or *irradiance* is integral of normal component of radiance over some solid angle:

$$F_\lambda = \int_\Omega I_\lambda \cos\psi d\Omega$$

where ψ is angle from normal to surface.

For example, the upwelling *hemispheric flux* on a horizontal plane is

$$F_\lambda^\uparrow = \int_0^{2\pi}\int_0^{\pi/2} I_\lambda \cos\theta\ \sin\theta d\theta d\phi \;\; = \int_0^{2\pi}\int_0^{1} I_\lambda(\mu,\phi)\ \mu\ d\mu d\phi$$

Flux = Spectral Power / normal Area. $\quad \mathrm{Units:}\;\; \mathrm{W\ m^{-2}\ \mu m^{-1}}$

Flux could be integral over field of view of detector, but usually over hemisphere.

Example: The normal incidence spectral solar flux at 0.5 μm at the orbit of the Earth is 1962 $\mathrm{W\ m^{-2}\ \mu m^{-1}}$. If the solar flux is converted to isotropic radiance with a reflective diffuser having 100% efficiency, what is the radiance?

Since isotropic radiance is independent of direction, the hemispheric flux is

$$F_\lambda = I_\lambda \int_0^{2\pi}\int_0^{1} \mu\ d\mu d\phi \; = \pi I_\lambda$$

Therefore the radiance is $I_\lambda = (1962\ \mathrm{W\ m^{-2}\ \mu m^{-1}})/\pi = 625\ \mathrm{W\ m^{-2}\ sr^{-1}\ \mu m^{-1}}$.

- Monochromatic *net flux* is the integral of the normal component of radiance over all solid angles: $F_{net,\lambda} = \int_{4\pi} I_\lambda \cos\theta d\Omega$

Net flux for a horizontal plane is the difference in upwelling and downwelling hemispheric flux:

$$F_{net,\lambda} = F_\lambda^\uparrow - F_\lambda^\downarrow \;=\; \int_0^{2\pi}\int_{-1}^{1} I_\lambda(\mu,\phi)\ \mu\ d\mu d\phi$$

Net flux is the net radiative energy flow in an atmosphere.

- *Actinic flux* is the total spectral energy at point (used in photochemistry):

$$F_{0,\lambda} = \int\int_{4\pi} I_\lambda(\Omega) d\Omega \quad = 4\pi \bar{I}_\lambda$$

  where $\bar{I}_\lambda$ is the mean intensity.

Spectral Integration

All radiative quantities may be spectrally integrated.
For example, the *broadband downwelling shortwave flux* is

$$F^{\downarrow}_{SW} = \int_{0.2\mu\text{m}}^{4.0\mu\text{m}} F^{\downarrow}_\lambda d\lambda$$

Monochromatic intensities and fluxes may be per wavelength or
per wavenumber. Intensity across a spectral interval must be the same: $I_\nu d\nu = I_\lambda d\lambda$

$$I_\nu = I_\lambda \left|\frac{d\lambda}{d\nu}\right| \quad = I_\lambda \frac{1}{\nu^2} \quad = I_\lambda \lambda^2$$

Example: Convert between radiance in per wavelength units to radiance in per wavenumber units at $\lambda = 10\ \mu\text{m}$:
Given $I_\lambda = 9.9\ \text{W m}^{-2}\ \text{sr}^{-1}\ \mu\text{m}^{-1}$. What is I_ν?

$I_\nu = (9.9\ \text{W m}^{-2}\ \text{sr}^{-1}\ \mu\text{m}^{-1})(10\ \mu\text{m}^{-1})(10^{-3}\ \text{cm}) \ = 0.099\ \text{W m}^{-2}\ \text{sr}^{-1}\ (\text{cm}^{-1})^{-1}$

Radiance vs. Flux

Constancy of intensity: If radiation is not interacting with matter then radiance is constant along a ray.
Solar flux depends on distance from sun (inverse square law):

$$F = F_\oplus \frac{r_\oplus^2}{r^2}$$

Flux decreases with distance squared because area of a sphere centered on the sun grows as r^2 and power crossing a sphere must be constant.
But intensity of solar radiation is constant because $I = F/\Omega$ and solid angle subtended by Sun decreases as $1/r^2$.

From an extended source, both radiance and flux are constant for a transparent medium. For example, the upward hemispheric flux from the moon's surface is constant with height because the solid angle subtended by the surfaces remains 2π until the curvature of the moon becomes important.

Extinction, Absorption, and Scattering

- **Extinction** - process of attenuation of a radiation beam by matter.
- **Absorption** - process of attenuation of radiation by conversion to another form of energy.
- **Scattering** - process of attenuation of radiation beam by redirecting light out of the beam.

Extinction = Absorption + Scattering

For example: Light from the Sun is *collimated*, meaning the rays are nearly parallel (because the angle subtended by the Sun is so small). Ultraviolet light in the collimated beam is 1) absorbed by ozone and ultimately converted to heat, and 2) scattered by air molecules into all directions via Rayleigh scattering. The UV flux in the solar beam is very much reduced (depending strongly on wavelength) before reaching the Earth's surface.

Emission of Radiation

Emission is the creation of radiation by matter. From quantum mechanics: atoms or molecules in excited states decay and the energy is converted to photons.

In thermodynamic equilibrium there is both thermal equilibrium and radiative equilibrium (emitted radiation equals absorbed radiation). This gives a particular distribution of excited states, and hence a particular radiation spectrum.

A *blackbody cavity* is a closed system in thermodynamic equilibrium, and has a particular emission spectrum that depends only on temperature: $I_\lambda = B_\lambda(T)$. This spectrum is called the Planck function.

It is called a blackbody because all radiation incident on cavity is absorbed. Blackbody cavities are important for calibration of IR instruments since they provide a known source of radiance.

- Blackbody radiation is isotropic and unpolarized.
- Blackbody radiation at a given wavelength depends only on temperature.

Planck Function

Planck derived $B_\lambda(T)$ by assuming quantized photon energies, $E = nhc/\lambda$.

See Thomas & Stamnes or appendix A in Liou or for derivation of Planck function.

Planck radiance function (per wavelength spectral interval):

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5[\exp(hc/k_B T\lambda) - 1]}$$

Factor of 2 in numerator because of 2 polarization modes emitted.

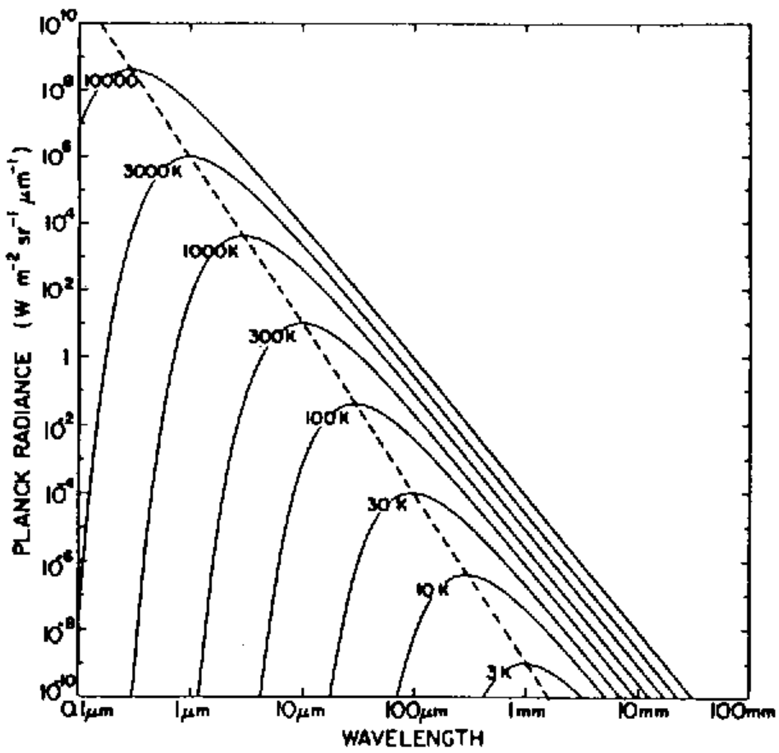
$$B_\lambda(T) = \frac{c_1}{\lambda^5[\exp(c_2/\lambda T) - 1]}$$

$$c_1 = 1.1911 \times 10^8 \text{ W m}^{-2} \text{ sr}^{-1} \, \mu\text{m}^4 \quad c_2 = 1.4388 \times 10^4 \text{ K } \mu\text{m}$$

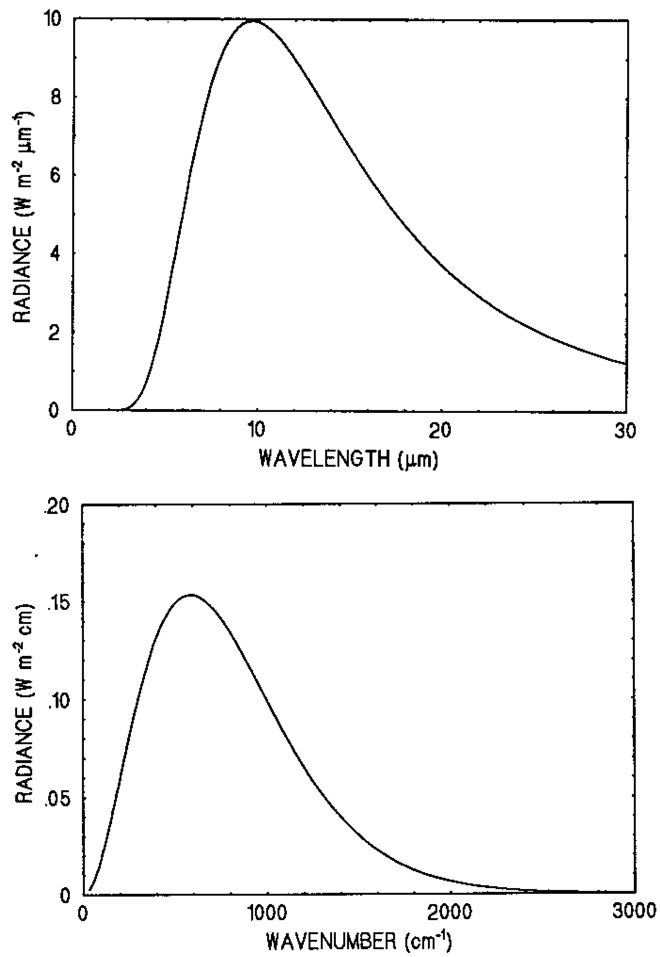
Planck function (per wavenumber spectral interval):

$$B_\nu(T) = \frac{c_1 \nu^3}{[\exp(c_2 \nu/T) - 1]}$$

$$c_1 = 1.1911 \times 10^{-8} \text{ W m}^{-2} \text{ sr}^{-1} \text{ cm}^4 \quad c_2 = 1.4388 \text{ K cm}$$



Planck functions on log-log plot for many temperatures.



Comparison of Planck function per wavelength and per wavenumber spectral intervals for $T = 300\,\mathrm{K}$.

Blackbody Radiation Laws

Wien's displacement law - maximum wavelength of Planck function is

$$\lambda_{\mathrm{max}} = \frac{2898\ \mu\mathrm{m\ K}}{T}$$

Stefan-Boltzmann Law - spectrally integrated *flux* from blackbody

$$F = \sigma T^4 \quad \sigma = 5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$$

Wiens limit - short wavelength limit.

$$B_\lambda(T) = \frac{c_1}{\lambda^5}\exp(-c_2/\lambda T)$$

Rayleigh-Jeans - long wavelength limit:

$$B_\nu(T) = 2k_B c\, \nu^2 T \qquad B_\lambda(T) = \frac{c_1}{c_2}\frac{T}{\lambda^4}$$

Radiance proportional to temperature in this limit.

Example: Compare the wavelength of maximum emission from the Sun (assume a blackbody of 5800 K) and a typical Earth surface (a blackbody at 290 K).

$$\lambda_{\text{max,Sun}} = \frac{2900\ \mu\text{m K}}{5800\ \text{K}} = 0.5\ \mu\text{m} \quad \text{mid visible}$$

$$\lambda_{\text{max,Earth}} = \frac{2900\ \mu\text{m K}}{290\ \text{K}} = 10\ \mu\text{m} \quad \text{mid infrared}$$

Example: Compare the flux emitted by a 290 K surface (e.g. the Earth) and a 5800 K surface (the Sun).

$$F_{\text{Earth}} = (5.67 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4})(290\ \text{K})^4 = 401\ \text{W m}^{-2}$$

$$F_{\text{Sun}} = (5.67 \times 10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4})(5800\ \text{K})^4 = 6.42 \times 10^7\ \text{W m}^{-2}$$

The ratio of flux emitted is $20^4 = 160,000$!

Brightness Temperature

Another way to express radiance - *brightness temperature* is the temperature of blackbody having the same radiance:

$$T_b = \frac{c_2}{\lambda \ln[1 + c_1/(I_\lambda \lambda^5)]}$$

Brightness temperature is very important for remote sensing, as it is often easier to interpret radiance in terms of temperature (see infrared spectrum figure).

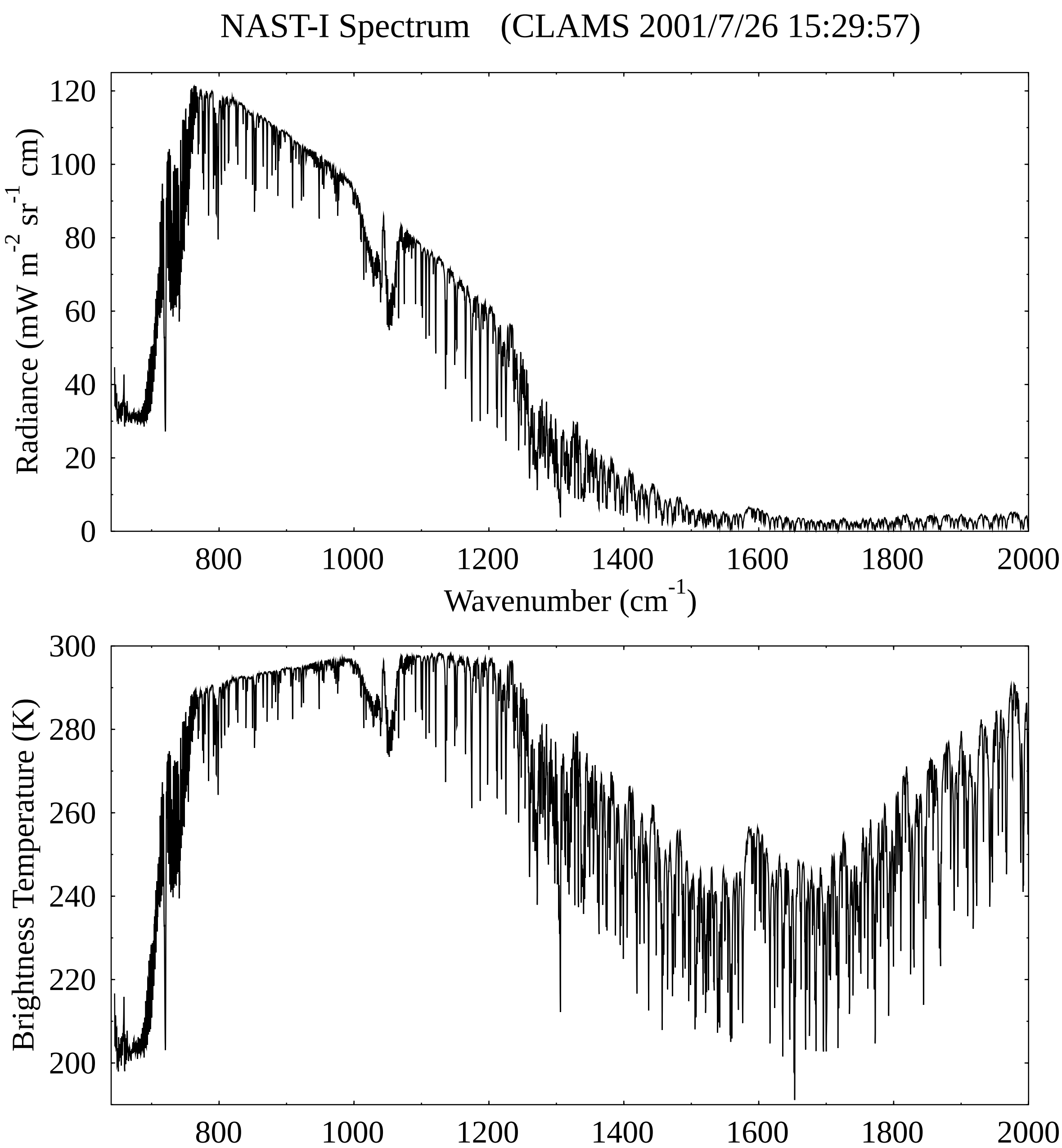
Spectral Integral of Planck Function

Integrals of the Planck function over spectral bands are needed for radiative flux calculations.
For $x = c_2\nu/T > 1$ the following series convergences rapidly

$$\int_\nu^\infty B_\nu(T) d\nu \ = c_1 \left(\frac{T}{c_2}\right)^4 \sum_{n=1}^{\infty} e^{-nx} \left[\frac{x^3}{n} + \frac{3x^2}{n^2} + \frac{6x}{n^3} + \frac{6}{n^4}\right]$$

For $x = c_2\nu/T < 1$ the following series convergences more rapidly

$$\int_0^\nu B_\nu(T)d\nu = c_1 \left(\frac{T}{c_2}\right)^4 \left[\frac{x^3}{3} - \frac{x^4}{8} + \frac{x^5}{60} - \frac{x^7}{5040} + \frac{x^9}{272160} - \cdots\right]$$



An example Earth upwelling infrared spectrum measured by the NAST-I interferometer spectrometer at 54,000 ft (0.25 cm^{-1} resolution). In the atmospheric window region around 10 μm the brightness temperature corresponds reasonable closely to the Earth's surface temperature.

Local Thermodynamic Equilibrium

In local thermodynamic equilibrium (LTE): **emission depends only on temperature and absorption properties of matter, not on the radiation field itself.**

In LTE:

- The time between quantum transitions from collisions
  $<<$ time between transitions from radiation.
- Boltzmann distributions apply to relevant atomic/molecular levels.
- Applies up to 50 to 80 km in Earth's atmosphere (depends on ν).

LTE applies in the Earth's troposphere and stratosphere, but not the mesosphere and thermosphere.
Thomas & Stamnes discuss LTE and also consider non-LTE, where the radiation field and population of quantum states are coupled. However, we will always assume LTE!

Emission from non-Blackbodies

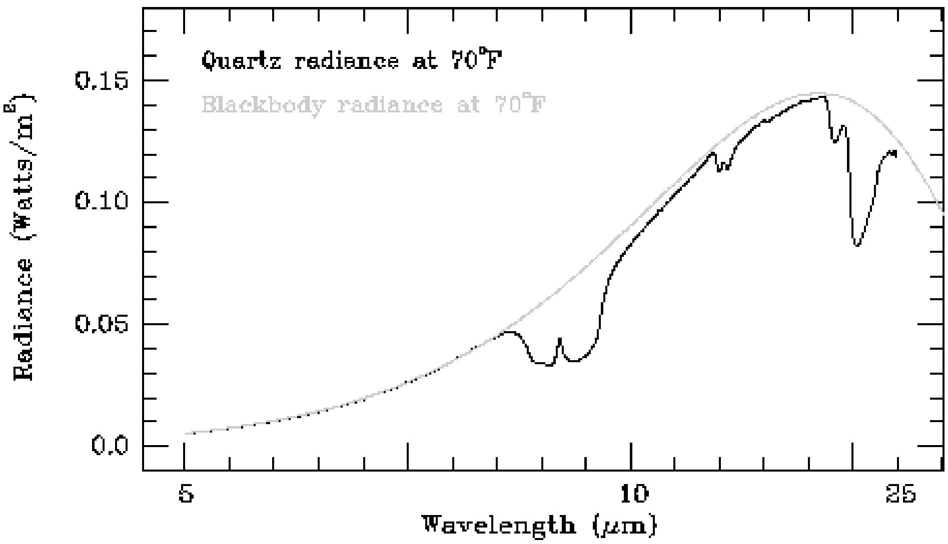
Emissivity = fraction emitted radiance is to that from a blackbody:
$\epsilon_\lambda = I_\lambda^{\text{emitted}}/B_\lambda$. Emitted radiance is thus

$$I_\lambda = \epsilon_\lambda B_\lambda(T)$$

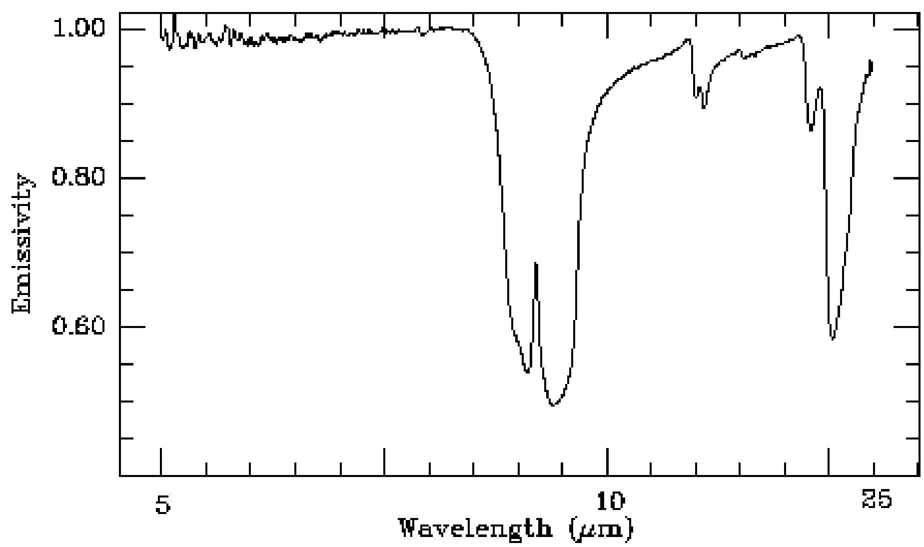
A blackbody has $\epsilon_\lambda = 1$.

Absorptivity = fraction of incident radiation that is absorbed (a_λ).

Kirchhoff's Law: emissivity equals absorptivity, $\epsilon_\lambda = a_\lambda$.
Derived from thermodynamic equilibrium, but also applies in LTE.



Quartz radiance spectrum along with a
blackbody radiance spectrum at the same temperature.



Quartz emissivity spectrum: the result of
dividing quartz radiance by blackbody radiance
at the same temperature.

Emitted radiance spectra for a blackbody and for a quartz surface (top). The quartz emissivity spectrum from dividing the two radiance spectra (bottom).