ATOC/ASTR 5560 Lab 2 Solutions
September 7, 2001

The purpose of this lab is to learn how to code multilayer thermal emission radiative transfer and understand how the resulting emergent radiance behaves. Log in to nit and copy the IDL file to your directory for this lab: `cp /home/rt/thermalrt/thermalrt.pro .` . *Turn in all the plots and a listing of the IDL file along with the answers to the questions.*

1. *Code the brightness temperature function at the beginning of the file.*

   See the IDL solution file on nit at `/home/rt/thermalrt/thermalrtsol.pro`.

2. *At the beginning of the IDL file are function definitions for upwelling and downwelling radiance calculations. We will assume the volume extinction has an exponential profile*

$$\beta(z) = \frac{\tau_{tot}}{z_0} e^{-z/z_0}$$

   where τ_{tot} is the total optical depth, z_0 is the extinction scale height, and z is the height above the surface. By integration this gives an an exponential optical depth profile:

$$\tau(z) = \tau_{tot} e^{-z/z_0}$$

   We will also assume that the atmospheric temperature is determined from the surface temperature T_s and the lapse rate Γ:

$$T(z) = T_s - \Gamma z$$

   Code the discrete multilayer thermal radiative transfer solutions for upwelling and downwelling radiance.

   Knowing the optical depth profile we can find the optical depth of each layer:

$$\Delta\tau = \tau_{tot} \left[e^{-z_{bot}/z_0} - e^{-z_{top}/z_0} \right]$$

   The multilayer radiative transfer solution can be done by iterating the single (isothermal) layer thermal emission solution:

$$I_1(\mu) = e^{-\Delta\tau/\mu} I_0(\mu) + [1 - e^{-\Delta\tau/\mu}] B_\lambda(T)$$

   where $I_1(\mu)$ is the radiance leaving one side of a layer at cosine zenith angle of μ, $I_0(\mu)$ is the radiance entering the layer on the other side, and $B_\lambda(T)$ is the Planck function at the layer temperature. The layer temperature can be obtained from the lapse rate using the mean layer height, $T(z_{middle}) = T_s - \Gamma(z_{bot} + z_{top})/2$.

   For upwelling radiance at the top of the atmosphere the iterations are started with the lowest layer. Since the surface is assumed to be black, there is no reflection of downwelling radiation. The incident radiation on the lowest layer is due the blackbody emission from the surface, $I_0(z = 0, \mu) = B_\lambda(T_{sfc})$.

   For downwelling radiance at the bottom of the atmosphere the iterations are started with the highest layer. There is no radiation incident from space, so $I_0(z = \infty, \mu) = 0$.

Note that the single layer solution is an approximation because the atmospheric layers are not actually isothermal. This is only a significant source of error for optically thick layers with a significant temperature gradient across them.

Test the code using $\lambda = 10.0\,\mu\text{m}$, $\mu = 0.8$, $T_s = 300\,\text{K}$, $\Gamma = 6.0\,\text{K/km}$, $\tau_{tot} = 1.0$, $z_0 = 2.0\,\text{km}$. *The upwelling radiance for this case is* $I_\lambda = 8.358\,\text{W m}^{-2}\,\text{sr}^{-1}\,\mu\text{m}^{-1}$ *and* $T_b = 289.7\,\text{K}$. *The downwelling radiance is* $I_\lambda = 6.180\,\text{W m}^{-2}\,\text{sr}^{-1}\,\mu\text{m}^{-1}$ *and* $T_b = 273.2\,\text{K}$.

3. *a) Use section* `Plotvstau` *to plot zenith upwelling intensity and brightness temperature at* $10.0\,\mu\text{m}$ *as a function of total atmosphere optical depth for three cases:*
i) $T_s = 300\,\text{K}$, $\Gamma = 6.0\,\text{K/km}$, $z_0 = 2.0\,\text{km}$ *ii)* $T_s = 300\,\text{K}$, $\Gamma = 6.0\,\text{K/km}$, $z_0 = 5.0\,\text{km}$
iii) $T_s = 300\,\text{K}$, $\Gamma = 0.0\,\text{K/km}$, $z_0 = 2.0\,\text{km}$

   b) Explain the dependence of radiance on total optical depth and the differences between the three cases.

   The upwelling radiance decreases with increasing optical depth for the atmospheric profiles having decreasing temperature. As the optical depth increases, the emission from the surface and lower atmosphere layers is obscured (little transmission), and the upwelling radiation comes from emission higher in the atmosphere. Since higher layers are colder in this atmosphere, the Planck function is less. Thus the high radiance from the warmer layer emission is replaced by lower radiance from the colder layer emission.

   Why don't the cold highest layers of the atmosphere cause the upwelling radiance to be very low? They normally have very little optical depth, so their emissivity is small and they transmit most of the upwelling radiation from below. So the upwelling radiance is emitted from a broad range where the transmission is less than one but above zero.

   The isothermal case has no dependence on optical depth because the Planck function is constant, regardless of the level in the atmosphere from which the emission originates.

   The larger scale height case gives lower upwelling radiance because the emission comes from higher (colder) in the atmosphere. The larger extinction scale height means that the moderate transmission region is a greater altitudes.

   c) For the first two cases calculate the heights and optical depths where the atmospheric temperature matches the brightness temperature for a total optical depth of 10. (Use IDL `print, taugrid` *to find the array location* `i` *for* $\tau = 10$ *and then use* `print, Tbup[i,0], Tbup[i,1]` *to get the brightness temperatures).*

   Using IDL print on the appropriate array location gives: i) $T_b = 267.8$ K and ii) $T_b = 226.6$ K. This is translated to height using the lapse rate

$$z = \frac{T_s - T}{\Gamma}$$

   resulting in average heights of emission of i) $z = 5.37\,\text{km}$ and ii) $z = 12.24\,\text{km}$. The heights are converted to optical depths with $\tau(z) = \tau_{tot} e^{-z/z_0}$, giving i) $\tau = 0.68$ and ii) $\tau = 0.86$.

   Even though the equivalent heights are quite different, the corresponding optical depths are similar, around $\tau = 1$. The thermal radiative transfer equation can be written with transmission as the independent variable (instead of optical depth) giving

$$I = \int_0^1 B_\lambda d\mathcal{T}$$

This shows, for example, that 80% of the emission weighting comes from where the transmission is between 0.1 and 0.9, which is between optical paths of 0.1 to 2.3.

4. *a) Use section* `Plotvsmu` *to plot upwelling and downwelling intensity and brightness temperature at* $10.0\ \mu\mathrm{m}$ *as a function of direction* μ *for the following case:*
   $\tau_{tot} = 1.0$, $T_s = 300\ \mathrm{K}$, $\Gamma = 6.0\ \mathrm{K/km}$, $z_0 = 2.0\ \mathrm{km}$.

   b) List the upwelling and downwelling brightness temperatures for $\mu = 1$ *and* $\mu = 0.3$. *Briefly explain the radiance behavior with* μ. *What is this effect called for the upwelling and downwelling cases?*

   Up $\mu = 1.0$ $T_b = 291.4\ \mathrm{K}$ $\mu = 0.3$ $T_b = 280.2\ \mathrm{K}$
   Down $\mu = 1.0$ $T_b = 266.6\ \mathrm{K}$ $\mu = 0.3$ $T_b = 293.1\ \mathrm{K}$
   Limb darkening for upwelling. Limb brightening for downwelling.

   The slant paths have longer optical paths, which means that the emission comes from height levels closer to the observer, as compared to straight ($\mu = 1$) viewing. For upwelling radiance the emission for slant paths originates from colder layers, hence limb darkening. For downwelling radiance the emission for slant paths originates from warmer layers, hence limb brightening.

5. *a) Use section* `ComputeFlux` *to calculate upwelling flux by integrating the radiance using Double-Gaussian quadrature. Compute the flux for* $\lambda = 10.0\ \mu\mathrm{m}$, $T_s = 300\ \mathrm{K}$, $\Gamma = 6.0\ \mathrm{K/km}$, $z_0 = 2.0\ \mathrm{km}$ *with* $\tau_{tot} = 1.0$ **and** $\tau_{tot} = 5.0$.

   The flux is calculated from the radiance at several angles using quadrature:

$$F = 2\pi \sum_j w_j \mu_j I(\mu_j)$$

   where μ_j are the quadrature angles and w_j are the quadrature weights. Using the $N_\mu = 4$ Double-Gauss quadrature gives
   $\tau_{tot} = 1.0$: $F = 25.10\ \mathrm{W\ m^{-2}} \mu\mathrm{m}^{-1}$ and $\tau_{tot} = 5.0$: $F = 18.31\ \mathrm{W\ m^{-2}} \mu\mathrm{m}^{-1}$.

   b) For the two cases in a) find the approximate $\bar{\mu}$ *that gives the same flux.*

   The single angle or *diffusivity approximation* is $F = \pi I(\bar{\mu})$, and $D = 1/\bar{\mu}$ is called the diffusivity factor. For upwelling flux for the two cases the best fitting angle is $\tau_{tot} = 1.0$: $\bar{\mu} = 0.59$ $\tau_{tot} = 5.0$: $\bar{\mu} = 0.62$. Both of these are close to $\bar{\mu} = 0.6$, and so a diffusivity factor of $D = 5/3$ is often assumed.