ATOC/ASTR 5560 — Test 2 Solutions
November 30, 2001
100 points total

1. (30 pts) *A volcanic eruption results in a layer of sulfuric acid aerosol droplets in the Earth's lower stratosphere. The layer is 5 km thick and has an average size distribution characterized by a lognormal distribution with* $N = 100\ \mathrm{cm}^{-3}$ *number concentration,* $r_0 = 0.2\ \mu\mathrm{m}$ *modal radius, and* $\sigma = 0.5$ *standard deviation of* $\ln r$*.*

   a) Calculate the effective radius of this size distribution. What is the size parameter of the effective radius for a wavelength of $\lambda = 10\ \mu$*m?*

   For a lognormal size distribution the effective radius (ratio of third to second moments) is

$$r_e = r_0 e^{5\sigma^2/2} \ = (0.2\ \mu\mathrm{m}) e^{2.5(0.5)^2} = 0.374\ \mu\mathrm{m}$$

   The size parameter for this radius is

$$e_e = \frac{2\pi r_e}{\lambda} \ = \frac{2\pi 0.374\ \mu\mathrm{m}}{10\ \mu\mathrm{m}} = 0.23$$

   b) Calculate the optical depth of the aerosol layer. The index of refraction of sulfuric acid at $\lambda = 10\ \mu$*m is* $m = 2.094 - 0.306i$ $(\frac{m^2-1}{m^2+2} = 0.542 - 0.0933i)$*.*

   The size parameter above indicates that the aerosol size distribution is close to the Rayleigh limit. Since the imaginary part of the index of refraction is large, the Rayleigh limit means that the absorption cross section will dominate the scattering cross section. The optical depth is the vertical integral of the volume extinction coefficient, and the extinction coefficient is the integral over the size distribution of the extinction cross section:

$$\tau = \int_0^\infty \beta_{ext} dz = \beta_{ext} \Delta z \qquad \beta_{ext} = \int_0^{infty} C_{abs} n(r) dr$$

   The Rayleigh absorption cross section formula is

$$C_{abs} = -\frac{8\pi^2 r^3}{\lambda} \mathrm{Im}\left[\frac{m^2-1}{m^2+2}\right]$$

   Combining these equations gives

$$\tau = \Delta z \frac{8\pi^2}{\lambda} \mathrm{Im}\left[\frac{m^2-1}{m^2+2}\right] \int_0^{infty} n(r) r^3 dr$$

   For a lognormal size distribution the third moment is

$$\int_0^{infty} n(r) r^3 dr = N r_0^3 e^{9\sigma^2/2}$$

   Therefore the optical depth is

$$\tau = 8\pi^2 (0.0933) \frac{5\ \mathrm{km}}{10\ \mu\mathrm{m}} (100\ \mathrm{cm}^{-3})(0.2\ \mu\mathrm{m})^3 (3.08)$$

$$\tau = (9.1 \text{ km cm}^{-3}\mu\text{m}^2)(10^5 \text{ cm/km})(10^{-4} \text{ cm}/\mu\text{m})^2 \ = 0.0091$$

c) If the temperature of the stratospheric aerosols is 215 K and the surface/lower atmosphere at 10 μm radiates like a blackbody at 290 K, what is the fractional change in upwelling longwave flux at 10 μm due to the volcanic aerosols?

$$B_\lambda(T) = \frac{c_1}{\lambda^5[\exp(c_2/\lambda T) - 1]} \quad c_1 = 1.1911{\times}10^8 \text{ W m}^{-2} \text{ sr}^{-1} \ \mu\text{m}^4 \quad c_2 = 1.4388{\times}10^4 \text{ K } \mu\text{m}$$

This part requires that you remember one of the most fundamental concepts from the first part of the course. The upwelling thermal infrared radiance from a single absorbing layer is

$$I = (1 - e^{-\tau/\mu})B_\lambda(T_a) + e^{-\tau/\mu}B_\lambda(T_s)$$

The first term is the emissivity times the Planck function at the aerosol temperature, while the second term is the transmissivity times the Planck function of the surface. To approximate flux from radiance at a single angle, choose $\mu = 0.6$. The transmissivity is

$$e^{-\tau/\mu} = e^{0.0091/0.6} = 0.985$$

The two Planck functions are

$$B_\lambda(T_a) = 1.48 \text{ W m}^{-2}\text{sr}^{-1}\mu\text{m}^{-1} \qquad B_\lambda(T_s) = 8.40 \text{ W m}^{-2}\text{sr}^{-1}\mu\text{m}^{-1}$$

The upwelling radiance at $\mu = 0.6$ including the aerosol layer is

$$I = 8.297 \text{ W m}^{-2}\text{sr}^{-1}\mu\text{m}^{-1}$$

This is to be compared with the radiance without the aerosol which is the Planck function at the surface. The aerosol layer decreases the upwelling flux by 1.2%. If this same percentage decrease was true across the longwave spectrum, then the global average decrease in outgoing longwave flux would be (0.012)(239 W/m^2)=3 W/m^2.

2. (20 pts) *Consider scattering of light by a large particle in the geometric optics limit.*

   a) What is the limiting value of the extinction efficiency Q_{ext}?

   $Q_{ext} = 2$

   If the imaginary part of the index of refraction m_i is zero, what is the value of the single scattering albedo ω?

   No absorption so $\omega = 1$.

   If the imaginary part of the index of refraction is significant ($|m_i| > 1/x$, where x is the size parameter), what is the value of the single scattering albedo ω in the limit $x \to \infty$?

   Rays that penetrate particle surface are all absorbed so $Q_{abs} = 1$ and $Q_{sca} = 1$ (from diffraction), so $\omega = 0.5$.

b) How does the width of the forward peak in the phase function scale with size parameter? Does the forward peak width depend significantly on the real or imaginary part of the index of refraction? Why or why not?

The width of the diffraction peak in the phase function is inversely proportional to size parameter, $\Delta\Theta_{peak} \propto 1/x$. The width does not depend on the index of refraction because diffraction is determined by the size of the particle, not composition, since the diffracted rays do not enter the particle. Fraunhoffer diffraction is derived by considering a hole in a screen, so it is the projected size and shape that matter.

c) The asymmetry parameter does not have a simple limiting value. The asymmetry parameter is about 0.85 for water droplets (for $r > 5$ μm) in the visible where the index of refraction is $m = (1.33, 0.0)$.

How does the limiting value (as $x \to \infty$) of the asymmetry parameter change as the imaginary part m_i increases in magnitude. Explain why.

g increases. As m_i increases there is more absorption, so a smaller fraction of the rays entering the particle escape. Thus the phase function becomes dominated by the diffraction peak in the $\Theta = 0$ direction and hence $g \to 1$.

How does the limiting value of the asymmetry parameter change as the real part m_r increases (assuming a small imaginary part). Explain why.

g decreases. As m_r increases the particle becomes more reflective (consider the Fresnel reflection formula) and more refractive. Thus penetrating rays are scattered by larger angles and hence g decreases.

3. (35 pts) *The dust aerosol in a planetary atmosphere has optical depth of 0.1, single scattering albedo of 0.84, and asymmetry parameter of 0.8 at 0.6 μm wavelength. At this wavelength the only other radiatively active component is molecular absorption with optical depth 0.02.*

a) Calculate the delta scaled **total** *atmosphere optical properties.*

First combine the optical properties:

$$\tau_t = \tau_a + \tau_g = 0.12 \qquad \omega_t = \frac{\tau_a \omega_a}{\tau_t} = 0.70$$

since the molecular absorption has $\omega_g = 0$.

The delta scaling formulas are

$$\tau' = (1 - \omega f)\tau \qquad \omega' = \frac{(1-f)\omega}{1 - \omega f} \qquad g' = \frac{g - f}{1 - f}$$

In the Eddington approximation we use $f = g^2$. The resulting scaled optical properties are

$$\tau' = 0.06624 \qquad \omega' = 0.4565 \qquad g' = 0.4444$$

b) Calculate the approximate atmosphere albedo assuming a black surface for a Sun angle of $\mu_0 = 0.5$.

Since the scaled optical depth is $\ll 1$ and flux quantities are desired, the optically thin limit of the Eddington solution for albedo can be used

$$R = \omega(\frac{1}{2} - \frac{3}{4} g\mu_0)\frac{\tau}{\mu_0}$$

$$R = (0.4565)[\frac{1}{2} - \frac{3}{4}(0.4444)(0.5)]\frac{0.06624}{0.5} = 0.0202$$

c) Calculate the direct and diffuse solar flux transmission at the surface for $\mu_0 = 0.5$. The direct flux is what would be measured by a narrow field of view Sun tracking pyrheliometer.

The direct flux transmission is obtained with Beer's law, and the unscaled optical depth must be used:

$$\mathcal{T}_{dir} = e^{-\tau/\mu_0} = e^{-0.12/0.5} = 0.7866$$

The total transmission is obtained from the Eddington optically thin limit

$$\mathcal{T}_{tot} = 1 - R - \frac{\tau'}{\mu_0}(1 - \omega') = 1 - 0.0202 - 0.0720 = 0.9078$$

The diffuse transmission is the difference

$$\mathcal{T}_{dif} = \mathcal{T}_{tot} - \mathcal{T}_{dir} = 0.9078 - 0.7866 = 0.1212$$

d) Calculate the surface albedo for which the atmosphere transitions from increasing the total (atmosphere + surface) albedo to decreasing the total albedo (compared with the surface alone).

The combined atmosphere + surface albedo R_t may be obtained from the adding formula, and we want to know for what surface albedo R_s the total albedo equals the surface albedo:

$$R_t = R_a + \frac{T_a^2 R_s}{1 - R_a R_s} = R_s$$

This is a quadratic equation in R_s

$$R_a R_s^2 - (1 - T_a^2)R_s + R_a = 0$$

with solution

$$R_s = \frac{1 - T_a^2 - \sqrt{(1 - T_a^2)^2 - 4R_a^2}}{2R_a}$$

The resuting surface albedo is $R_s = 0.116$, above which the atmosphere reduces the albedo.

4. (15 pts) *Give a one or two sentence description of the following terms:*

a) Discrete Dipole Approximation

A numerical method for calculating scattering from a nonspherical particle. It involves dividing the particle up into regions (dipoles) small compared to the wavelength and computing the interaction among all the dipoles.

b) Eigen-matrix method

A numerical method for solving the plane-parallel discrete ordinate matrix radiative transfer equation using eigenvalues and eigenvectors. This is the method DISORT uses.

c) Nakajima-Tanaka method for accurate radiances

A method for computing accurate radiances from the delta-M scaled radiative transfer equation. The single scattered radiation is calculated exactly using the exact phase function, while the higher order terms come from the radiative transfer solution using the delta-M Legendre series truncated phase function.

d) Fresnel reflection

Specular refelction from a plane dielectric surface. The reflection coefficients depends on the index of refraction and the incident angle.

e) Independent pixel approximation

A radiative transfer method for dealing with inhomogeneities in the optical depth field, say of overcast clouds. The plane-parallel radiative transfer results are integrated over the optical depth distribution.