

ATOC/ASTR 5560 Test 1 Solutions

October 12, 2001

1. (30 pts) Calculate the upwelling radiance ($\text{W m}^{-2} \text{sr}^{-1} \mu\text{m}^{-1}$) at zenith angle of 30° from a dusty Martian atmosphere at a wavelength of $10 \mu\text{m}$. The dust layer has a volume extinction coefficient that is linear from 0.01 km^{-1} at the surface to zero at 20 km. The dust does not scatter mid-infrared radiation. The average (extinction weighted) temperature of the dust layer is 230 K. The surface emits like a blackbody with a temperature of 250 K. Calculate the brightness temperature corresponding to this radiance.

The optically thin atmosphere can be approximated by a single isothermal layer at the effective atmosphere temperature, $T_a = 230 \text{ K}$. We first need the atmosphere optical depth. Since the extinction profile is linear, the optical depth is the average extinction times the layer thickness

$$\tau = \int_0^{20} \beta dz = \bar{\beta} \Delta z = (0.005 \text{ km}^{-1})(20 \text{ km}) = 0.10$$

The transmission is

$$\mathcal{T} = e^{-\tau/\cos\theta} = e^{-0.1/0.866} = 0.891$$

The single layer radiative transfer solution is

$$I_\lambda = e^{-\tau/\mu} B_\lambda(T_s) + (1 - e^{-\tau/\mu}) B_\lambda(T_a)$$

The Planck function is

$$B_\lambda(T) = \frac{c_1}{\lambda^5 [\exp(c_2/\lambda T) - 1]}$$

which for the surface and atmosphere temperatures at $\lambda = 10 \mu\text{m}$ is

$$B(250 \text{ K}) = 3.783 \text{ W m}^{-2} \text{sr}^{-1} \mu\text{m}^{-1} \quad B(230 \text{ K}) = 2.291 \text{ W m}^{-2} \text{sr}^{-1} \mu\text{m}^{-1}$$

The upwelling radiance at $\theta = 30^\circ$ is then

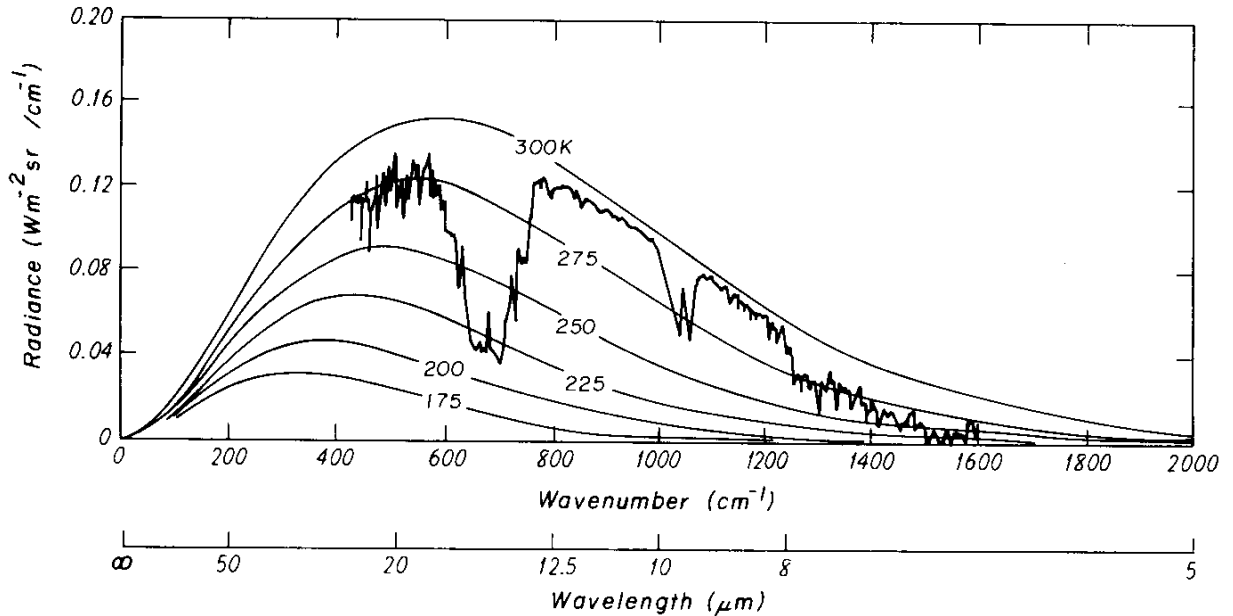
$$I_\lambda = (0.891)(3.783) + (1 - 0.891)(2.291) = 3.621 \text{ W m}^{-2} \text{sr}^{-1} \mu\text{m}^{-1}$$

The brightness temperature corresponding to this radiance is

$$T_b = \frac{c_2}{\lambda \ln[1 + c_1/(I_\lambda \lambda^5)]} = 248.1 \text{ K}$$

which is close to, but colder than the surface temperature for this optically thin atmosphere.

2. (30 pts) Below is a clear sky radiance spectrum taken of the Earth from space.



a) Write down the appropriate radiative transfer solution for this situation. Identify the terms.

We can express the upwelling radiance for midinfrared thermal emission using weighting functions as

$$I_\nu(\infty, \mu) = \mathcal{T}_\nu(\infty, 0)I_\nu(0, \mu) + \int_0^\infty B_\nu[T(z)] W_\nu(z, \infty) dz$$

where the weighting function is

$$W_\nu(z, \infty) = \left| \frac{d\mathcal{T}_\nu(z, \infty)}{dz} \right|$$

and $\mathcal{T}_\nu(z, \infty)$ is the transmission at wavenumber ν from level z to the top of the atmosphere. $B_\nu[T(z)]$ is the Planck function at level z which is at temperature $T(z)$. $I_\nu(0, \mu)$ is the upwelling radiance at the surface, which is close to that of a blackbody at the surface temperature. The weighting function thus gives the contribution, or weighting of the Planck profile, to the upwelling radiance from level z .

b) Explain the major features of brightness temperature from 400 to 1400 cm^{-1} . Use the weighting function concept. Consider the spectral variation in absorption coefficient and the distribution of absorbing gases and temperature.

The brightness temperatures can be obtained by interpolating the radiance spectrum between the blackbody curves. There are five major features in the spectrum. The region from 400 to 600 cm^{-1} has a brightness temperature around 275 K. This is the upper end of the water

vapor pure rotational band, in which the absorption by water vapor is moderate. The transmission to the surface is near zero, but the transmission from space to the upper troposphere is near one. Thus the weighting function is peaking in the midtroposphere at temperatures around the freezing point of water (273 K).

The region from 600 to 800 cm^{-1} is dominated by the CO_2 ν_2 vibrational-rotational absorption band. The brightness temperature mainly decreases towards the band center as the absorption increases due to increasing line strength. The increasing absorption reduces the transmission and therefore raises the weighting function to higher altitudes. Since temperature decreases in the troposphere the brightness temperature decreases. However, in the center of the band the brightness temperature *increases* because the weighting function peaks in the upper stratosphere where the temperature is warmer than the tropopause.

The region from 800 to 1200 cm^{-1} is the atmospheric window (except for 1000 to 1100 cm^{-1}). The absorption by the atmosphere is very low, except for some continuum water vapor absorption in the lowest few km. Thus, the weighting function peaks at the surface and only has contributions from the lowest few km of the atmosphere. The upwelling brightness temperature is around 295 K, since the lower atmosphere and surface are warm (the surface would appear to have a temperature around 300 K) and the emissivity of the surface is near unity.

The 1000 to 1100 cm^{-1} region shows a decrease in brightness temperature to around 270 K due to the 9.6 μm ozone band. The ozone is in the stratosphere and concentrated in the lower stratosphere, where the temperatures are below 225 K. The fact that the brightness temperature is not that cold indicates that ozone layer is not opaque, since some of the radiation upwelling from the lower troposphere is transmitted through the ozone layer. Thus, the weighting function has a substantial contribution from the surface in addition to a peak in the stratosphere.

The last region is from 1200 to 1400 cm^{-1} which is part of the 6.3 μm water vapor vibrational band. There the brightness temperature decreases as the weighting function peak altitude increases with increasing absorption towards the band center. The brightness temperatures are again indicative of midtroposphere temperatures.

3. (40 pts) *The simplest band model is one that assumes the spectrum has **isolated, strong** absorption lines. This model is appropriate when there are equally spaced strong lines and the line width is suitably narrow.*

a) Derive an expression for the mean transmission across a band of width $\Delta\nu$ from space down to pressure p . The band contains strong Lorentz lines with strengths S_i and line halfwidths $\alpha_{0,i}$ at reference pressure p_0 . Ignore the temperature dependence, i.e. assume that the line parameters S_i and $\alpha_{0,i}$ are tabulated for the appropriate temperature. Assume the absorbing gas is well mixed and has a mass mixing ratio of q . Use the scaling approximation for the inhomogeneous path (both the scaling approximation and the Curtis-Godson approximation are exact for the strong line limit with an isothermal atmosphere). Show that the band mean transmission is linear in pressure p .

Since the absorption lines are isolated we can simply add up the equivalent widths from all

the absorption lines. The equivalent width of a single Lorentz line in the strong limit is

$$W_i = 2\sqrt{S_i\alpha_i u}$$

where S_i is the line strength, α_i is the line halfwidth, and u is the absorber amount. Therefore the band mean transmission is

$$\mathcal{T}_{\Delta\nu} = 1 - \frac{1}{\Delta\nu} \sum_i W_i = 1 - \frac{2}{\Delta\nu} \sum_i \sqrt{S_i\alpha_i u}$$

Since we are assuming a constant temperature, the line strengths S_i are constant. The line halfwidth varies along the path due to the changing pressure. This can be dealt with using the scaling approximation for the absorber amount

$$\tilde{u} = \int \frac{p}{p_0} \rho_a dz$$

Since the absorber, with density ρ_a , is a well mixed gas we can use the hydrostatic relation, $\rho_a dz = -q dp/g$, to get

$$\tilde{u} = \frac{q}{g} \int_0^p \frac{p}{p_0} dp = \frac{qp^2}{2gp_0}$$

The regular absorber amount from the hydrostatic relation is $u = qp/g$, so the scaling simply reduces the absorber amount by a factor of $2p/p_0$. Alternatively, one can use the Curtis-Godson approximation to scale the halfwidth

$$\bar{\alpha} = \frac{\alpha_0}{u} \int \frac{p}{p_0} du = \frac{\alpha_0 p}{2p_0}$$

This means that one simply uses the line halfwidth at the average pressure of the path. You can see that the scaling approximation for the absorber amount u and the Curtis-Godson approximation give the same result, which is

$$\mathcal{T}_{\Delta\nu} = 1 - \frac{2}{\Delta\nu} \sum_i \sqrt{S_i \alpha_{0,i} \frac{qp^2}{2gp_0}}$$

or

$$\mathcal{T}_{\Delta\nu} = 1 - \frac{2}{\Delta\nu} \left(\sum_i \sqrt{S_i \alpha_{0,i}} \right) \sqrt{\frac{q}{2gp_0}} p$$

This is linear in pressure as required.

b) Calculate the band mean transmission from 625 to 665 cm^{-1} from space to 10 mb in Earth's atmosphere from CO_2 which has a mass mixing ratio of $q = 5.6 \times 10^{-4}$. The line parameters at $p_0 = 1013 \text{ mb}$ and $T = 230 \text{ K}$ are $\sum_i S_i = 27400 \text{ cm/g}$ and $\sum_i \sqrt{S_i \alpha_{0,i}} = 489 \text{ g}^{-1/2}$.

From part a the band mean transmission is

$$\mathcal{T}_{\Delta\nu} = 1 - \frac{2}{\Delta\nu} \left(\sum_i \sqrt{S_i \alpha_{0,i}} \right) \sqrt{\frac{qp^2}{2gp_0}}$$

The scaled absorber amount is

$$\tilde{u} = \frac{qp^2}{2gp_0} = \frac{(5.6 \times 10^{-4})(1000 \text{ Pa})}{2(9.8 \text{ m/s}^2)} \frac{10 \text{ mb}}{1013 \text{ mb}} = 0.000282 \text{ kg/m}^2 = 2.82 \times 10^{-5} \text{ g/cm}^2$$

For the band parameter given the mean transmission is

$$\mathcal{T}_{\Delta\nu} = 1 - \frac{2}{40 \text{ cm}^{-1}} (489 \text{ g}^{-1/2}) \sqrt{2.82 \times 10^{-5} \text{ g/cm}^2} = 1 - 0.130 = 0.870$$

Since the mean transmission is linear in pressure, it is easy to find the band transmission down to any pressure level

$$\mathcal{T}_{\Delta\nu} = 1 - 0.013p/(1 \text{ mb})$$

The LBLRTM and simple band model transmissions at a range of pressures are:

Pressures (mb)	LBLRTM Transmission	Isolated strong line transmission
0 - 30	0.7176	0.610
0 - 10	0.8917	0.870
0 - 3.0	0.9630	0.961
0 - 1.0	0.9841	0.987
0 - 0.3	0.9925	0.996
0 - 0.10	0.9960	0.9987

c) Why does this band model fail at higher pressures?

Clearly, this formula for band mean transmission must fail at higher pressures, since it is negative above 77 mb. The reason is that the line width increases with pressure so that the lines are no longer isolated. The absorption lines overlap, which reduces the total equivalent width or absorptance, and the transmission is thus lower than the model calculates.

d) This band model fails at low pressures because it is missing another physical mechanism. Explain.

This band model assumes that the absorption line width scales linearly with pressure. In reality, the lines stop decreasing in width with pressure because their width becomes dominated by Doppler broadening at very low pressures. Thus the true mean transmission does not change linearly with pressure, but more slowly (if all the lines remained strong, then the absorption would go as \sqrt{p}).