ATOC/ASTR 5560 Lab 8 Solutions

November 2, 2001

The purpose of this lab is to learn about simple solar radiative transfer approximations, specifically the first order scattering solution and the Eddington two-stream model. Log in to nit and copy the following files to your directory:

`/home/rt/simplert/simplert.pro`	*IDL file for the lab*
`/home/rt/simplert/readscatfile.pro`	*IDL procedure to read miegamma scattering files*

1. *Code the first order scattering radiative transfer solution in section* `FirstOrder` *of the IDL file. Use the linear in optical depth form of the first order solution. Calculate the upwelling radiance as a function of zenith angle for a solar flux $S_0 = 1$. Review the IDL code to see how the scattering angle is calculated from the solar and outgoing directions and how the phase function is calculated. Write down the line you coded in IDL:*

   ```
   radiance = solarflux*ssalbedo*phasefunc/(4*!pi) *tau/cos(theta)
   ```

   Plot the first order solution radiances for the mineral aerosol case in Lab 8. You will need the scattering file from `miegamma.f` *for this case. The radiances are plotted versus zenith angle in the solar plane (for convenience the $\phi = 180°$ direction is plotted with negative zenith angles). Use an aerosol optical depth of 0.15 and compute the radiance field for two sun angles: $\theta_0 = 0°$ and $\theta_0 = 60°$.*

   What is the layer thickness (km) for this aerosol distribution corresponding to an optical depth of 0.15?

   The thickness corresponding to an optical depth of 0.15 is found from the aerosol extinction

   $$\Delta z = \frac{\tau}{\beta} = \frac{0.15}{0.321 \text{ km}^{-1}} = 0.467 \text{ km}$$

   If the solar flux is unity ($S_0 = 1$), what are the units of radiance?

   The units of radiance are flux per solid angle, so radiance units in this case are ster^{-1}. That is one reason the radiance values are so small. The other reasons are, of course, the low optical depth and forward scattering phase function.

   In radiative transfer terms, why does the radiance increase for large zenith angles.

   The viewing direction intersects a longer path length for oblique outgoing zenith angles. This is manifested in the τ/μ term of the first order solution.

   Explain the differences between the radiance plots for the two sun angles.

   For overhead sun there is increased radiance around an outgoing zenith angle of $0°$. This is due to the backscatter peak in the phase function. The radiance pattern is symmetric around $\theta = 0$. For the $60°$ sun angle, the backscatter peak is shifted to the $-60°$ direction, where it is twice the radiance value due to the longer path. There is a large increase in radiance for the positive zenith angles, which is due to the increase in the phase function in the forward scattering direction.

2. *The Eddington two-stream solution for flux reflectance (albedo) and transmittance from Meador and Weaver (1980) has been coded in IDL for you. Use section* `Eddington` *in the IDL file and the special case solutions in the notes to check 1) the albedo and absorptance in the optically thin limit, and 2) the albedo in the conservative scattering limit. Choose your own parameters for a few cases and give the results below (don't bother with delta-Eddington yet).*

I used $\tau = 0.02$, $\omega = 0.8$, $g = 0.85$, $\mu_0 = 0.5$ for the optically thin limit.
IDL Eddington: $R = 0.00567 \quad A = 0.00797$
Equations: $R = 0.00580 \quad A = (1-\omega)\tau/\mu_0 = 0.008$
The albedo difference is 2.3%. The solutions should converge as $\tau \to 0$.

For the conservative scattering case, putting in an $\omega = 1$ causes the Meador and Weavor solution to fail due to divide by zero. This is because $\omega = 1$ is a special case where the eigenvalue $k = 0$, so the exponentials in the solution are no longer valid. The best thing to do in this case is to make ω very close, but not equal to 1.

I chose parameters $\tau = 5$, $\omega = 0.99999$, $g = 0.85$, $\mu_0 = 0.5$.
IDL Eddington: $R = 0.43996 \quad A = 0.00009$
Conservative scattering equation: $R = 0.44000 \quad A = 0$

3. *A highly accurate multi-stream radiative transfer model has been run for the $\lambda = 2.13$ μm, $r_{eff} = 10$ μm cloud cases in Lab 7. Below is a table with results for albedo R and absorptance A for different optical depths and sun angles. The results are for a single homogeneous layer atmosphere above a black surface.*

τ	ω	g	μ_0	R_{MS}	A_{MS}
1.0	*0.97854*	*0.843*	*1.0*	*0.04717*	*0.02387*
10.	*0.97854*	*0.843*	*1.0*	*0.29783*	*0.31510*
1.0	*0.97854*	*0.843*	*0.25*	*0.32230*	*0.07568*
10.	*0.97854*	*0.843*	*0.25*	*0.56047*	*0.27518*

Compare the **Eddington** *and the* **delta-Eddington** *albedo and absorptance with the multi-stream results. List percentage errors.*

τ	μ_0	R	A	R_{Edd}	A_{Edd}	$R_{\delta Edd}$	$A_{\delta Edd}$
1.0	1.0	0.04717	0.02387	-0.03836	0.02980	0.04604	0.02402
10.	1.0	0.29783	0.31510	0.25361	0.37614	0.29849	0.31055
1.0	0.25	0.32230	0.07568	0.36396	0.04537	0.27787	0.06442
10.	0.25	0.56047	0.27518	0.56323	0.24019	0.53892	0.27196

		Percent Errors			
τ	μ_0	R_{Edd}	A_{Edd}	$R_{\delta Edd}$	$A_{\delta Edd}$
1.0	1.00	-181	24.8	-2.4	0.6
10.0	1.00	-14.8	19.4	0.2	-1.4
1.0	0.25	12.9	-40.1	-13.8	-14.9
10.0	0.25	0.5	-12.7	-3.8	-1.2

For what range of parameters do the Eddington and delta-Eddington approximations appear to perform worse.

The straight Eddington approximation is particularly poor for low optical depths for mostly forward scattering phase functions ($g > 0.5$). The delta-Eddington approximation is accurate accept for high solar zenith angles at low optical depths. In general the delta-Eddington approximation is a big improvement over the unscaled Eddington solution.

4. *Make plots of the* **delta-Eddington** *albedo and absorptance versus optical depth using section* `PlotEddington`.

   a) Consider the effect of single scattering albedo: use $\omega = 0.9999, 0.999, 0.99$ with $g = 0.85$ and $\mu_0 = 1.0$.

   b) Consider the effect of sun angle: use $\mu_0 = 1.0, 0.866, 0.5$ with $\omega = 0.99$ and $g = 0.85$.

   Describe the behavior of the reflection and absorption in the two optical depth limits ($\tau \to 0$ and $\tau \to \infty$) for the $\mu_0 = 1$ plot.

   For small optical depths ($\tau \to 0$) the albedo is linear in optical depth and the three single scattering albedo cases converge to the same albedo. The absorptance is also linear in this limit, but the slope is proportional to ($1 - \omega$). For large optical depths ($\tau \to \infty$) the albedo and absorptance saturate to a constant, more quickly when there is more absorption. The $\tau \to \infty$ reflectance decreases with decreasing ω, but a single scattering albedo near one ($\omega = 0.99$) has a much lower asymptotic albedo than the conservative case. In the infinite optical depth limit the transmission is zero, so $A = 1 - R$.

   Explain the results in terms of multiple scattering and the scattering geometry.

   The albedo increases linearly with optical depth at first, because for small optical depth the first order scattering solution is applicable. Even though there must be multiple scattering for $\tau > 1$, it is mainly in the forward direction and does not contribute much to the reflection, so the linearity of the first order solution remains applicable. As the optical depth increases further the albedo increases more slowly. For large optical depth even a very small amount of single scattering absorption ($1 - \omega$) causes a significant absorptance. This is due to multiple scattering amplifying the single scattering absorption; the absorptance goes as $1 - \omega^n$ where n is the average number of scatterings. The number of orders of scattering increases with optical depth. The albedo for $\omega = 0.99$ saturates for $\tau > 30$ because the absorption causes very little energy to diffuse down to the bottom of the layer, and thus adding more optical depth would not reflect more flux.

   There is more reflection for larger solar zenith angle (smaller μ_0). The photons injected into the top of the layer stay closer to the top, and thus have a greater chance to escape if the solar zenith angle is large. The absorptance for larger solar zenith angle is greater for small τ due to the longer path, but is lower for large τ due to the shallower photon penetration.