ATOC/ASTR 5560 Lab 6 Solutions

October 19, 2001

The purpose of this lab is to learn about the behavior of Rayleigh scattering from single particles and learn about particle size distributions. Log in to nit and copy the following files to your directory:

`/home/rt/rayleigh/rayleigh.pro`	*IDL lab file*
`/home/rt/rayleigh/shettle.dat`	*aerosol index of refraction file*
`/home/rt/rayleigh/mie_aerosol_w10.dat`	*Mie scattering results*

1. *Code the expressions for Rayleigh absorption and scattering cross sections in the IDL file in section* `plot_rayleigh_accuracy`.

   The Rayleigh cross sections in IDL are:

```
Csca1 = (128/3.)*pi^5 * (r^6/lambda^4) * abs( (m1^2-1)/(m1^2+2) )^2
Cabs1 = -8*pi^2 * r^3/lambda * imaginary( (m1^2-1)/(m1^2+2) )
Csca2 = (128/3.)*pi^5 * (r^6/lambda^4) * abs( (m2^2-1)/(m2^2+2) )^2
Cabs2 = -8*pi^2 * r^3/lambda * imaginary( (m2^2-1)/(m2^2+2) )
```

2. *Plot the Rayleigh absorption and scattering cross sections and the Mie cross sections as a function of particle radius. The Mie cross sections are computed for aerosols composed of ammonium sulfate and sea salt at a wavelength of 10 μm. Look in* `shettle.dat` *for the index of refraction of these aerosols at this wavelength.*

   At a wavelength of $10\ \mu$m the index of refraction of ammonium sulfate is $m = 2.190 - 0.130i$ and for sea salt is $m = 1.540 - 0.015i$.

   At what radii do the Rayleigh absorption and scattering cross sections diverge from the Mie results by 10% (for each case)?
   (for example use `print, r[where(abs(Csca1/Csca1mie-1) gt 0.10)]`.

   At 10 μm wavelength the Rayleigh 10% error radius for ammonium sulfate is 0.465 μm for absorption and 0.780 μm for scattering. For sea salt the 10% error radius is 0.695 μm for absorption and 1.705 μm for scattering.

   Absorption is more sensitive to Mie effects due to Q_{abs} being linear in Mie coefficients a_n, b_n, while Q_{sca} is quadratic in a_n, b_n. The aerosol with the smaller index of refraction stays in the Rayleigh regime longer because the size of the Mie coefficients depends on the index of refraction.

3. *Graph the same Rayleigh-Mie comparison on a log-log plot (use* `/xlog, /ylog` *in the* `plot` *command). Explain the slopes of the lines. At what radius does the Mie scattering cross section reach the absorption cross section for each aerosol?*

   The Rayleigh cross sections are straight lines on a log-log plot because they are power laws. The absorption cross section slope is 3 ($C_{abs} \propto r^3$) and the scattering cross section slope is 6 ($C_{sca} \propto r^6$).

   Use `print, r[where(Csca1mie/Cabs1mie lt 1)]` in IDL. The Mie scattering cross section reaches the absorption cross section at $r = 0.975$ μm for ammonium sulfate and $r = 0.79$ μm for sea salt. There is more absorption for ammonium sulfate (Im[m]) so it takes a larger radius for scattering to catch up, even though there is also more scattering for ammonium sulfate (Re[m]).

4. *Use section* `plot_distribution` *to make plots of the log normal particle* **number** *and* **volume** *distributions with i)* $N_{tot} = 100\ \mathrm{cm}^{-3}$, $r_0 = 0.2$ μm, $\sigma = 0.40$, *and ii)* $N_{tot} = 100\ \mathrm{cm}^{-3}$, $r_0 = 0.1$ μm, $\sigma = 0.70$. *The volume distribution is the distribution of particle volumes as a function of radius (*$V(r)$ *instead of* $n(r)$*). The IDL section makes the plots in terms of linear and log radii.*

   First you will need to code the lognormal distribution formula in the IDL function, and then make the volume distributions from the number distributions in `plot_distribution`.

   What is the radii of the peaks in the number and volume distributions (use `print, max(dist1,i), r[i]`*)?*

   The lognormal size distribution formula coded in IDL is

```
n_r = N/(sqrt(2*!pi)*sigma) * (1/r) * exp(-alog(r/r0)^2 /(2*sigma^2)
```

   The volume distribution is the volume of each particle size times the concentration at that radius:

$$V(r) = \frac{4\pi r^3}{3} n(r)$$

   The peaks in the distributions are at $r_{max}^{(1)} = 0.170$ μm and $r_{max}^{(2)} = 0.062$ μm for number concentration, and $r_{max}^{(1)} = 0.276$ μm and $r_{max}^{(2)} = 0.266$ μm for volume distribution.

   Even though the second distribution has half the r_0, the volume distribution peaks are almost the same. This is because the smaller r_0 distribution is also wider (larger σ) so there are more large particle which contribute more to the volume.

5. *Use the* `integrate_rayleigh` *section of the IDL file to numerically integrate the Rayleigh absorption cross sections to find the volume absorption coefficient β_{abs}. Use the first size distribution for the ammonium sulfate aerosols and the second size distribution for the sea salt aerosols. Convert the units to km^{-1}.*

   The volume absorption coefficient is the integral of the absorption cross section over the size distribution:

$$\beta_{abs} = \int_0^\infty C_{abs}(r) n(r) dr$$

   When implementing this on a computer remember to put in the dr in the size distribution integration:

$$\beta_{abs} = \sum_i C_{abs}(r_i) n(r_i) \Delta r$$

   For example, in IDL `print, total(Cabs1*dist1)*radmax/Nrad`. The units of cross section are in microns (since λ and r are in μm). Therefore units of integral come out in $\mu\mathrm{m}^2/\mathrm{cm}^3$ and must be multiplied by $(10^{-4}\ \mathrm{cm}/\mu\mathrm{m})^2(10^5\ \mathrm{cm}/\mathrm{km})$ to get km^{-1}.

   The absorption coefficients are:
   $\beta_{abs} = 4.8 \times 10^{-4}\ \mathrm{km}^{-1}$ for the sulfate aerosol, size distribution 1, and
   $\beta_{abs} = 5.1 \times 10^{-5}\ \mathrm{km}^{-1}$ for the sea salt aerosol, size distribution 2.

   In actuality these hygroscopic aerosol would most likely be hydrated, and the addition of water would swell the aerosols causing greater extinction. Nevertheless this illustrates that most aerosols in the mid-infrared have very low optical depths.