ASTR/ATOC 5560 Problem Solving Solutions Week 9

1. *Calculate the upwelling radiance in direction* $\theta = 45°$ *and* $\phi = 90°$ *at wavelength* $\lambda = 0.85\ \mu$*m from an aerosol layer of optical depth* $\tau = 0.15$*. The single scattering albedo is 0.88 and the phase function may be approximated by a Henyey-Greenstein phase function with* $g = 0.70$*. The direction* **towards** *the Sun is* $\theta = 45°$ *and* $\phi = 180°$*. The solar flux is about* $1000\ \mathrm{W\ m^{-2}}\mu\mathrm{m}^{-1}$ *at this wavelength.*

   The aerosol optical depth is small compared to 1 so we can use the first order scattering solution for radiance

   $$I_1(\mu, \phi) \ = S_0 \frac{\omega P(\Theta)}{4\pi} \frac{\tau}{\mu}$$

   where S_0 is the TOA solar flux, $\mu = \cos\theta$ is the cosine of the viewing zenith angle, ω is the single scattering albedo, and τ is the optical depth. The phase function $P(\Theta)$ is obtained in this problem from the asymmetry parameter g by assuming a Henyey-Greenstein phase function

   $$P_{HG}(\Theta) = \frac{1 - g^2}{(1 + g^2 - 2g\cos\Theta)^{3/2}}$$

   The scattering angle Θ is a function of the solar and viewing geometries:

   $$\cos\Theta = \cos\theta\cos\theta' + \sin\theta\sin\theta'\cos(\phi' - \phi)$$

   Note: the incident direction (θ', ϕ') is the opposite direction from the Sun: $\theta' = 180 - \theta_0 = 135°$ and $\phi' = \phi_0 - 180 = 0°$. Therefore the cosine of the scattering angle is

   $$\cos\Theta = \cos(45°)\cos(135°) + \sin(45°)\sin(135°)\cos(0 - 90°) = -0.5 \quad (\Theta = 120°)$$

   The phase function at this scattering angle is

   $$P(\Theta) = \frac{1 - 0.7^2}{[1 + 0.7^2 - 2(0.7)(-0.5)]^{3/2}} \ = 0.1574$$

   The upwelling radiance is

   $$I_1(\mu, \phi) \ = (1000\ \mathrm{W\ m^{-2}}\mu\mathrm{m}^{-1}) \frac{(0.88)(0.1574)(0.15)}{4\pi(0.707)} = 2.34\ \mathrm{W\ m^{-2}sr^{-1}}\mu\mathrm{m}^{-1}$$