ASTR/ATOC 5560 Problem Solving Solutions Week 8

1. *In the microwave region, liquid water droplets are in the Rayleigh regime. Calculate the Rayleigh backscattering coefficient that would be observed by a cloud radar operating at* $\lambda = 8.66mm$ *for a lognormal size distribution of water droplets with* $N = 150cm^{-3}$*,* $r_0 = 10\mu m$ *and* $\sigma = 0.35$*. The Rayleigh backscattering coefficient is given by* $\sigma_{back} = \beta_{sca}P(180°)$*. For water at* $10°$*,* $K = \frac{m^2-1}{m^2+2} = -0.495 - 0.059i$

   The Rayleigh phase function is

$$P(180°) = \frac{3}{4}(1 + cos^2(180°)) = 1.5$$

   The Rayleigh volume scattering coefficient for a particle size distribution is

$$\beta_{sca} = \int_0^\infty \pi r^2 Q_{sca} n(r) dr$$

   where

$$Q_{sca} = \frac{8}{3}x^4 \left|\frac{m^2-1}{m^2+2}\right|^2 = \frac{8}{3}\left(\frac{2\pi r}{\lambda}\right)^4 (0.2485)$$

$$\left|\frac{m^2-1}{m^2+2}\right|^2 = |K|^2 = (0.495^2 + 0.059^2) = 0.2485$$

   Therefore

$$\beta_{sca} = (.2485)\frac{8}{3}\left(\frac{2\pi}{\lambda}\right)^4 \pi \int_0^\infty r^6 n(r) dr$$

   For a lognormal distribution

$$\int_0^\infty r^6 n(r) dr = N r_0^6 \exp(18\sigma^2)$$

   So the volume scattering coefficient is

$$\beta_{sca} = (.2485)\frac{8}{3}\left(\frac{2\pi}{\lambda}\right)^4 \pi N r_0^6 \exp(18\sigma^2) \quad (1)$$

$$= (0.2485)\frac{128\ \pi^5\ 150cm^{-3}}{3\ (0.866cm)^4}(0.001cm)^6 \exp(2.205) \quad (2)$$

$$= 7.85 \times 10^{-7} km^{-1} \quad (3)$$

   Therefore the Rayleigh backscattering coefficient is

$$\sigma_{back} = P(180)\beta_{sca} = (1.5)(7.85 \times 10^{-7} km^{-1}) = 1.18 \times 10^{-8} km^{-1}$$

2. *Explain the differences between the m=1.33 and m=1.55 Qext vs x curves on page 7 of the notes. Consider the* $x < 1$ *region and the* $x > 2$ *regions separately, and refer to the equations of the appropriate simple theory for each case.*

   The $x < 1$ region is in the Rayleigh regime. The index of refraction, m, is real so there is no absorption and the extinction efficiency is equal to the scattering efficiency. The Rayleigh formula for scattering efficiency is

$$Q_{sca} = \frac{8}{3}x^4 \left|\frac{m^2-1}{m^2+2}\right|^2$$

   For m=1.33, $K^2 = \left|\frac{m^2-1}{m^2+2}\right|^2$ = 0.0416 and for m=1.55, $K^2 = 0.101$. Therefore, for the same size parameter, x, the m=1.55 case will have a larger scattering efficiency than the m=1.33 case in the Rayleigh regime.

	Q_{sca} at given x	
m	x=0.5	x=0.9
1.33	0.007	0.073
1.55	0.017	0.504

   For the $x > 2$ region, use the anomalous diffraction theory (ADT). In this theory for non-absorbing spheres,

$$Q_{ext} = 2 - \frac{4}{\rho}\sin\rho + \frac{4}{\rho^2}(1-\cos\rho),$$

   where $\rho = 2x(m-1)$ is the phase lag.

   The first maximum in the extinction efficiency should be at $\rho \sim 4.1$. Solving for the size parameter, $x = \frac{\rho}{2(m-1)}$.

m	x at maximum
1.33	6.2
1.55	3.7

   The locations of the rest of the max/min are also expressed by the theory. For m=1.33, the phase lag, ρ, is smaller because there is less difference between the index of refraction of the particle and that of air. Since the phase lag is smaller, the difference in size parameter between each max/min is larger.