ASTR/ATOC 5560 Problem Solving Solutions Week 6

1. *In the table below are the k-distribution values for Fu and Liou's band 16 from 400 to 540 cm^{-1}. The k-distribution values have been interpolated in pressure and temperature to the midpoints of the two layers in the midlatitude winter atmosphere. The table also contains the absorber amount u of water vapor in each layer. Calculate the band mean zenith transmission from 0 to 4 km.*

Layer (km)	u (kg/m^2)	k_1 (m^2/kg)	k_2	k_3	k_4	k_5	k_6	k_7
0 - 2	2.58	0.148	0.246	0.49	1.38	6.02	46.8	198
2 - 4	1.20	0.101	0.170	0.35	0.98	4.40	38.5	189
Δg_j		0.12	0.24	0.24	0.20	0.12	0.06	0.02

Using a k-distribution the band mean transmission is found by a weighted sum of transmissions (from τ_j) for each k:

$$\mathcal{T}_{\Delta\nu} = \sum_{j=1}^{N} \Delta g_j \exp\left(- \sum_{l=1}^{L} k_j(z_l) \Delta u_l \right)$$

For the first k,

$$\tau_1 = k_1(0-2)u_{0-2} + k_1(2-4)u_{2-4} \ = 0.148(2.58) + 0.101(1.20) = 0.503$$

	1	2	3	4	5	6	7
τ_j	0.503	0.839	1.684	4.74	20.8	167	738
$e^{-\tau_j}$	0.605	0.432	0.186	0.009	0.0	0.0	0.0
Δg_j	0.12	0.24	0.24	0.20	0.12	0.06	0.02

The band mean transmission for the two layers is

$$\mathcal{T}_{\Delta\nu} = \sum_{j=1}^{7} \Delta g_j \ e^{-\tau_j} \ = 0.223$$

2. *Extra busy work: Calculate the band mean transmissions for each layer separately and show that Beer's Law does not apply to band mean transmission.*

Applying the same procedure to find the transmission for each k of the two layers separately gives

j	1	2	3	4	5	6	7
$\mathcal{T}_j(0-2)$	0.683	0.530	0.282	0.028	0.0	0.0	0.0
$\mathcal{T}_j(2-4)$	0.886	0.815	0.657	0.309	0.005	0.0	0.0

The resulting two layer mean transmissions are $\mathcal{T}_{\Delta\nu}(0-2) = 0.283$ and $\mathcal{T}_{\Delta\nu}(2-4) = 0.522$. If Beer's Law applied to band mean transmission then the product of the two layer transmissions would equal the total transmission of the two layers. But $\mathcal{T}_{\Delta\nu}(0-2)\,\mathcal{T}_{\Delta\nu}(2-4) = 0.148 \;<\; 0.223 = \mathcal{T}_{\Delta\nu}(0-4)$. One could multiply layer band transmissions if the absorption was uncorrelated, but the water vaper absorption lines are highly correlated between the two layers, which allows more radiation to be transmitted.