ASTR/ATOC 5560 Problem Solving Solutions Week 5

1. *Below is a profile of broadband longwave upwelling and downwelling flux calculated by MDTERP for a standard midlatitude summer atmosphere. Calculate the net flux at levels from 10 to 15 km. Then calculate the layer mean net flux divergence (W/m^3) and heating rate (K/day) for the layer from 10 to 11 km and the layer from 13 to 14 km.*

The net flux is the difference between the upwelling and downwelling flux:

$$F_{net} = F^{\uparrow} - F^{\downarrow}$$

The results are in the table.

Height (km)	Pressure (mb)	Temp (K)	Density (g/m^3)	Fup (W/m^2)	Fdown (W/m^2)	Fnet (W/m^2)
0.0	1013	294	1201	423.53	348.21	75.32
1.0	902	290	1084	410.41	304.65	105.77
2.0	802	285	981	393.50	264.06	129.44
3.0	710	279	887	375.42	225.79	149.63
4.0	628	273	802	359.31	191.94	167.37
5.0	554	267	723	345.58	162.41	183.17
6.0	487	261	650	333.40	136.35	197.05
7.0	426	255	582	322.36	112.46	209.89
8.0	372	248	523	312.71	91.06	221.65
9.0	324	242	466	304.50	71.81	232.69
10.0	281	235	417	297.58	53.40	244.18
11.0	243	229	370	292.14	37.51	254.63
12.0	209	222	328	287.96	27.50	260.45
13.0	179	216	289	284.81	23.00	261.81
14.0	153	216	247	282.78	21.04	261.74
15.0	130	216	210	281.42	19.49	261.93

The net flux divergence for the layer can be calculated by finite differencing the net flux across the layer

$$\frac{dF_{net}}{dz} = \frac{F_{net}(z_2) - F_{net}(z_1)}{z_2 - z_1}$$

For the 10 to 11 km layer the net flux divergence is

$$\frac{dF_{net}}{dz} = \frac{254.63\ \mathrm{W/m^2} - 244.18\ \mathrm{W/m^2}}{1000\ \mathrm{m}} = 0.0105\ \mathrm{W/m^3}$$

This is positive indicating that energy is leaving the layer and so it is cooling. For the 13 to 14 km layer the net flux divergence is

$$\frac{dF_{net}}{dz} = \frac{261.74\ \mathrm{W/m^2} - 261.81\ \mathrm{W/m^2}}{1000\ \mathrm{m}} = -7 \times 10^{-5}\ \mathrm{W/m^3}$$

This is very slightly negative, indicating heating.

The heating rate is obtained from the net flux convergence by dividing by the heat capacity per volume of air

$$\left.\frac{dT}{dt}\right|_{rad} = -\frac{1}{\rho C_p}\frac{dF_{net}}{dz} = \frac{g}{C_p}\frac{dF_{net}}{dp}$$

It is more accurate to use the pressure form (so density is not interpolated). For the 10 to 11 km layer the heating rate is

$$\left.\frac{dT}{dt}\right|_{rad} = \frac{9.8\ \mathrm{m/s^2}}{1004\ \mathrm{J\ kg^{-1}K^{-1}}}\frac{254.63\ \mathrm{W/m^2} - 244.18\ \mathrm{W/m^2}}{24300\ \mathrm{Pa} - 28100\ \mathrm{Pa}}$$

$$\left.\frac{dT}{dt}\right|_{rad} = -2.68\times 10^{-5}\ \mathrm{K/s}\ (86400\ \mathrm{s/day}) = -2.32\ \mathrm{K/day}$$

The negative heating rate indicates cooling.
For the 13 to 14 km layer the heating rate is

$$\left.\frac{dT}{dt}\right|_{rad} = \frac{9.8\ \mathrm{m/s^2}}{1004\ \mathrm{J\ kg^{-1}K^{-1}}}\frac{261.74\ \mathrm{W/m^2} - 261.81\ \mathrm{W/m^2}}{15300\ \mathrm{Pa} - 17900\ \mathrm{Pa}}$$

$$\left.\frac{dT}{dt}\right|_{rad} = -2.6\times 10^{-7}\ \mathrm{K/s}\ (86400\ \mathrm{s/day}) = 0.02\ \mathrm{K/day}$$

2. *a) Calculate the net flux divergence in the 400 to 500 cm^{-1} spectral band for the layer from 10 to 11 km in the midlatitude summer atmosphere using the* cooling to space approximation. *Compare with the net flux divergence obtained from the net fluxes in the table below (calculated with MODTRAN3 using the diffusivity approximation). The table also contains the flux transmissivities to space and the band integrated Planck functions.*

z *(km)*	p *(mb)*	T *(K)*	ρ *(g/m^3)*	$F_{net,400-500}$ *(W m^{-2})*	$\mathcal{T}^f_{400-500}(z,\infty)$	$B_{400-500}(T)$ *(W m^{-2}sr^{-1})*
10.0	*281*	*235*	*417*	*28.13*	*0.9044*	*7.334*
11.0	*243*	*229*	*370*	*29.02*	*0.9535*	*6.793*

The cooling to space term of the heating rate equation is the product of the Planck *flux* emission, integrated over the spectral band, and the vertical gradient of the flux transmission:

$$\left.\frac{dF_{\Delta\nu,net}}{dz}\right|_{\text{space}} = \pi B_{\Delta\nu}(z)\frac{\partial \mathcal{T}^f_{\Delta\nu}(z,\infty)}{\partial z}$$

The derivative of the transmission is the weighting function referenced to space. This derivative may be done with a finite difference across the layer using the mean Planck function value to represent the layer average temperature:

$$\left.\frac{dF_{\Delta\nu,net}}{dz}\right|_{\text{space}} = \pi(7.064\ \mathrm{W\ m^{-2}\ sr^{-1}})\frac{0.9535 - 0.9044}{1000\ \mathrm{m}} = 1.09\times 10^{-3}\ \mathrm{W/m^3}$$

For comparison the net flux divergence from the net fluxes on each side of the layer is

$$\frac{dF_{\Delta\nu,net}}{dz} = \frac{F_{\Delta\nu,net}(z_2) - F_{\Delta\nu,net}(z_1)}{\Delta z} = 0.89 \times 10^{-3}\ \mathrm{W/m^3}$$

Thus the cooling to space approximation is fairly accurate for this layer and spectral region. It slightly overestimates the cooling, because the layer is heated a little by flux exchange with adjacent layers. The flux exchange with the surface is zero because there is no transmission to the surface due to strong water vapor absorption in the pure rotation band of water vapor.

b) Make the comparison between the total net flux divergence and the cooling to space approximation for the same layer, but for the 980 to 1080 cm^{-1} spectral band.

z (km)	p (mb)	T (K)	ρ (g/m^3)	$F_{net,980-1080}$ (W m^{-2})	$\mathcal{T}^f_{980-1080}(z,\infty)$	$B_{980-1080}(T)$ (W m^{-2}sr^{-1})
10.0	281	235	417	21.51	0.5944	2.387
11.0	243	229	370	21.17	0.6022	2.024

Using the same procedure as before the flux divergence in this spectral band for the layer calculated with the cooling to space net approximation is

$$\left.\frac{dF_{\Delta\nu,net}}{dz}\right|_{\mathrm{space}} = \pi(2.206\ \mathrm{W\ m^{-2}\ sr^{-1}})\frac{0.6022 - 0.5944}{1000\ \mathrm{m}} = 5.4 \times 10^{-5}\ \mathrm{W/m^3}$$

For comparison the net flux divergence from the net fluxes on each side of the layer is

$$\frac{dF_{\Delta\nu,net}}{dz} = \frac{21.17\ \mathrm{W/m^2} - 21.51\ \mathrm{W/m^2}}{1000\ \mathrm{m}} = -3.4 \times 10^{-4}\ \mathrm{W/m^3}$$

The cooling to space approximation fails miserably in this case, since there significant net flux *convergence* (heating). This spectral region is the 9.6 μm ozone band. There is little ozone in the lower troposphere so the warm surface and lower tropospheric water vapor emission can reach the upper troposphere and lower stratosphere where it is absorbed, causing heating. The cooling to space is largely blocked by the ozone in the stratosphere above.