**ASTR/ATOC 5560 Problem Solving Solutions Week 3**

1. *a) What altitude is at the triple point of water (273 K) in the standard tropical atmosphere, midlatitude winter atmosphere, and the atmosphere of Venus?*

   Reading off the graphs: tropical $z_f = 5$ km midlat winter $z_f = 0$ km
   Venus $z_f = 58$ km

   *b) Estimate the mass mixing ratio of water vapor in a standard tropical atmosphere at the surface and at the tropopause.*

   At the surface the mass mixing ratio is $q = \rho_v/\rho = 19/1200 = 0.016$. The tropopause in the standard tropical atmosphere is at 17 km, where the mass mixing ratio is $q = 5 \times 10^{-4}/170 = 3$ ppm.

2. *The fundamental vibration of CO occurs at 2143 cm$^{-1}$. The rotational constant of CO is 1.93 cm$^{-1}$. Consider a transition in which the vibrational quantum number changes from $v = 0$ to $v = 1$ and the rotational number changes from $J = 11$ to $J = 10$.*

   *a) Is this transition emission or absorption?*

   The vibrational energy change (which is positive) is much larger than the rotational energy change (which is negative), so the transition absorbs energy.

   *b) What branch is this transition in? (P, Q, R)*

   The rotational transition has $\Delta J = -1$ so this is the P branch, meaning wavenumbers below the band center.

   *c) What is the wavenumber of this transition?*

   The energy of the molecule has two parts, vibrational and rotational.

$$E = E_{vib} + E_{rot}$$

   The vibrational change in energy is

$$\Delta E_{vib}/hc = 2143 \text{ cm}(1 - 0) = +2143 \text{ cm}$$

   The rotational energy from the rigid rotator model is

$$E_{rot}/hc = J(J + 1)B$$

   so the rotational energy change is

$$\Delta E_{rot} = [10(10 + 1) - 11(11 + 1)]B = (110 - 132)B = -22B = -42.5 \text{ cm}$$

   The net change in energy and hence the spectral position of the line is

$$\nu = 2143 \text{ cm} - 42.5 \text{ cm} = 2100.5 \text{ cm}$$

3. *What is the spectral range of the atmospheric window in the midinfrared (4 to 20 $\mu$m) in the Earth's atmosphere?*

   About 8 to 12 $\mu$m, except for the notch at 9.6 $\mu$m from the ozone band.

   *What spectral range do you expect for the atmospheric window in the Martian atmosphere? Why?*

   There is very little water vapor on Mars and no ozone, so the midinfrared atmospheric window would be between the 4.3 and 15 $\mu$m $CO_2$ bands. There is 30 times more $CO_2$ on Mars than Earth, so you would expect the $CO_2$ absorption bands to be somewhat wider (though the colder temperatures will tend to make them narrower). So an estimate of the spectral range of the window on Mars is from 5 to 13 $\mu$m.

4. *Explain the difference in the 6.3 $\mu$m band origin wavenumber for the isotopes of water vapor. Explain the differences in the sum of the strengths across the band $S_n$.*

   Classically the frequency of a harmonic oscillator (e.g. mass on a spring) depends inversely with the mass, $\omega_0 = \sqrt{k/m}$, i.e., the heavier the mass, the lower the oscillation frequency. This carries over to quantum mechanics where the oscillator is the atoms in a molecule. The different isotopes have the same electronic configuration, and hence the same bond strength, but have different reduced masses. Therefore, the heavier isotopes have a fundamental vibration energy that is lower than the lighter isotope (however, the change is not quite $\sqrt{k/m}$).

   The different isotopes have the same absorption coefficient per molecule, but the tabulated line strengths include the isotopic abundances in the Earth's atmosphere. Thus the rare isotopes of water have much lower line strengths because there are so few of them relative to the main isotope of water.

5. *What is the total range in percent of the solar flux during the course of a year at the top of the atmospheres of Venus, Earth, and Mars?*

   The distance between the Sun and a planet changes from $r_0(1-e)$ to $r_0(1+e)$ during the year, where $r_0$ is the mean distance and $e$ is the eccentricity (assuming $e \ll 1$). The solar flux at the planet goes as the inverse square of the distance from the Sun or $F \propto 1/r^2$. Therefore the total fractional range in solar flux is

$$\frac{(1+e)^2}{(1-e)^2} - 1 \approx 4e$$

| | Eccentricity | $\frac{(1+e)^2}{(1-e)^2}$ | Flux Change |
|---|---|---|---|
| Venus | 0.007 | 1.028 | 2.8% |
| Earth | 0.017 | 1.070 | 7.0% |
| Mars | 0.093 | 1.45 | 45% |