ASTR/ATOC 5560 Problem Solving Week 14

1. *Use the single layer gray energy balance model to compute the radiative equilibrium temperature of the Earth's surface and atmosphere for the case where the atmosphere absorbs some solar radiation. Assume 1) the atmosphere is a single isothermal layer at temperature T_a, 2) the atmosphere has a longwave emissivity of ϵ, a shortwave absorptivity of a, and shortwave reflectivity of r, 3) the surface is black in the longwave and shortwave and has a temperature T_s.*

   a) Derive expressions for the global annual average surface and atmospheric temperatures in terms of the solar constant S, and the shortwave and longwave atmospheric properties.

   The energy balance at the top of the atmosphere is that the incident solar flux is equal to the reflected solar flux plus the outgoing longwave flux from the atmosphere and surface:

$$\text{TOA}: \quad \frac{S}{4} = \frac{S}{4}r + \epsilon\sigma T_a^4 + (1-\epsilon)\sigma T_s^4$$

   The energy balance at the surface is that the transmitted fraction $1 - r - a$ of solar flux plus the longwave flux emitted by the atmosphere equals the longwave flux emitted by the surface:

$$\text{Surface}: \quad (1-r-a)\frac{S}{4} + \epsilon\sigma T_a^4 = \sigma T_s^4$$

   These two equations may be added to eliminate the atmospheric temperature term

$$(2-2r-a)\frac{S}{4} = (2-\epsilon)\sigma T_s^4$$

   The surface temperature can then be found from

$$\sigma T_s^4 = \left(\frac{2-2r-a}{2-\epsilon}\right)\frac{S}{4}$$

   The atmosphere layer temperature is found by substituting the surface temperature in one of the energy balance equations. After some manipulation this results in

$$\sigma T_a^4 = \left(\frac{a+\epsilon(1-a-r)}{\epsilon(2-\epsilon)}\right)\frac{S}{4}$$

   b) Calculate the surface and atmospheric temperatures for a longwave emissivity of 0.8. Assume a shortwave absorptivity of 0.1 and use the Earth's current global average albedo for the atmospheric reflectivity.

   For atmospheric parameters $\epsilon = 0.8$, $a = 0.1$, and $r = 0.3$, (and solar constant $S = 1366\ \mathrm{W/m^2}$) the single layer model temperatures are

$$T_s = 284\ \mathrm{K} \quad T_a = 246\ \mathrm{K}$$

c) Imagine that a full scale nuclear war ignites and burns enough petroleum and forests so that the soot in the atmosphere absorbs the sunlight and none reaches the surface. Say the planetary albedo drops to 0.05, but the longwave emissivity doesn't change. Calculate the surface and atmosphere temperatures.

No solar flux reaches the surface so the solar absorption must be $a = 0.95$. For $\epsilon = 0.8$, $a = 0.95$, and $r = 0.05$, the single layer model temperatures are

$$T_s = 263 \text{ K} \quad T_a = 278 \text{ K}$$