ASTR/ATOC 5560   Problem Solving Solutions   Week 13

1. *Calculate the day length and mean daily solar TOA insolation for the June 21 solstice at 65°N latitude. Assume the true anomaly is linearly proportional to time.*

   The solar declination on the summer solstice is equal to the obliquity of the Earth, which is $\delta = \epsilon = 23.5°$. The day length is obtained from the solar declination and the latitude $\phi = 65°$:

   $$\cos H = -\tan\phi\tan\delta = -(0.435)(2.145) = -0.932 \qquad H = 137°(-43.0° + 180°)$$

   Day length is $(24 \text{ hr})H/180° = 18.3 \text{ hrs}$

   June 21 is about 169 days after the January 3 perihelion. With the linear in time assumption the true anomaly is

   $$\nu \approx 360(169/365.24) = 166.6°$$

   The ratio of the Earth-Sun distance to the mean distance is then

   $$\frac{r}{r_0} \approx 1 - e\cos\nu = 1 - (0.017)\cos(166.6) = 1.0165$$

   The daily averaged solar flux is

   $$\bar{F} = \frac{S_0}{\pi}\left(\frac{r_0}{r}\right)^2 (H\sin\phi\sin\delta + \sin H\cos\phi\cos\delta)$$

   In radians $H = 2.39$, so the TOA daily mean solar flux is

   $$\bar{F} = S_0/\pi\ (1.0165)^{-2}[2.39\sin 65\sin 23.5 + \sin 137\cos 65\cos 23.5]$$

   $$\bar{F} = (1366 \text{ W/m}^2)(0.968)(1.128)/\pi = 475 \text{ W/m}^2$$

2. *Calculate the mean daily solar insolation for the December 21 solstice at 65°S latitude. Compare with the result in 1.*

   At the other solstice the solar declination is $\delta = -23.5°$. Since we are now using a latitude of $\phi = -65°$, the day length will be the same as in 1. The only thing that is different for the daily mean solar flux is the Earth-Sun distance.

   December 13 is about 13 days before the January 3 perihelion. With the linear in time assumption the true anomaly is

   $$\nu \approx 360(-13/365.24) = -12.8°$$

   The ratio of the Earth-Sun distance to the mean distance is then

   $$\frac{r}{r_0} \approx 1 - e\cos\nu = 1 - (0.017)\cos(-12.8) = 0.983$$

   The daily mean TOA solar flux is then

   $$\bar{F} = (1366 \text{ W/m}^2)(1.034)(1.128)/\pi = 507 \text{ W/m}^2$$

   This is 32 W/m^2 or about 7% higher than the situation for the other hemisphere and solstice.