ASTR/ATOC 5560 Problem Solving Solutions Week 12

1. *We have an atmosphere made of geometrically thin clouds of optical depth 10 occupying 25% of the horizontal area embedded in a uniform haze of optical depth 1. Assume both the haze and the cloud do not absorb at visible wavelengths and both have an asymmetry parameter of $g = 0.8$. Assume the surface is black.*

   a) Use the Independent Pixel Approximation to calculate the domain averaged albedo for a solar angle of $\mu_0 = 2/3$ (use the Eddington solution).

   Since the clouds are geometrically thin, there is little leakage of photons out the clouds sides, so 3D finite cloud effects are small. Therefore, the main effect of the horizontal inhomogeneity is due to the nonlinear relation between optical depth and albedo. This is dealt with by using the independent pixel approximation.

   The independent pixel approximation for domain averaged albedo is to integrate over the probability distribution of optical depth $p(\tau)$.

$$R_{ipa} = \int p(\tau) R_{pp}(\tau) d\tau$$

   For this problem there are only two possible optical depths: the haze covering 75% and the cloud plus haze covering 25%. Thus the IPA albedo is

$$R_{ipa} = (1-f)R_{pp}(\tau = 1) + fR_{pp}(\tau = 11)$$

   where $f = 0.25$ is the cloud fraction.

   We can use the Eddington conservative scattering albedo for R_{pp}, which for $\mu_0 = 2/3$ is

$$R_{pp} = \frac{(1-g)\tau}{4/3 + (1-g)\tau}$$

   The albedo of the haze is $R_{haze} = 0.130$ and the albedo of the cloud plus hase is $R_{cloud} = 0.623$ Therefore

$$R_{ipa} = 0.75(0.130) + 0.25(0.623) = 0.253$$

   b) Compare the result in part a) with the plane-parallel result (using the mean optical depth).

   The average optical depth is $\tau_{avg} = 0.75(1.0) + 0.25(11.0) = 3.5$. The plane-parallel albedo for the mean optical depth is

$$R_{pp} = \frac{(1-0.8)(3.5)}{4/3 + (1-0.8)(3.5)} = 0.344$$

   As expected, the plane-parallel albedo is considerably greater than the true domain averaged albedo obtained by the independent pixel approximation.