ASTR/ATOC 5560 Problem Solving Solutions Week 11

1. *Say a water cloud has an optical depth of 10 and asymmetry parameter of 0.85 at visible and near IR wavelengths. Use the Eddington solution to obtain the albedo of the cloud for a solar zenith angle of $\mu_0 = 2/3$.*

   Now suppose the cloud is above a grassy surface. Calculate the total albedo (cloud and surface, ignoring the small contribution from molecular scattering) at a wavelength of $\lambda = 0.65$ μm where the surface albedo is 0.10 and at $\lambda = 0.85$ μm where the surface albedo is 0.65. Use the two-stream approximation so the reflection matrix is treated as a scalar. What is the multiple reflection factor in each case.

   At wavelengths of 0.65 μm and 0.85 μm water does not absorb, so this is conservative scattering. For a solar angle of $\mu_0 = 2/3$ the Eddington conservative scattering albedo is

$$R = \frac{(1-g)\tau}{4/3 + (1-g)\tau} = \frac{(1-0.85)10}{4/3 + (1-0.85)10} = 0.529$$

   Since there is no absorption the transmission is $T = 1 - R = 0.471$.

   The cloud layer and surface may be combined radiatively using the adding formula:

$$R_T = R_a + T_a \Gamma R_s T_a \qquad \Gamma = [1 - R_s R_a]^{-1}$$

   where R_a is the atmosphere reflection, T_a is the atmosphere transmission, R_s is the surface reflection, and Γ is the multiple reflection factor.

   For $\lambda = 0.65$ μm the total albedo is

$$R_{T,0.65} = 0.529 + \frac{(0.471)(0.10)(0.471)}{1 - (0.10)(0.529)} = 0.529 + \frac{0.022}{0.947} = 0.553$$

   For $\lambda = 0.85$ μm the total albedo is

$$R_{T,0.85} = 0.529 + \frac{(0.471)(0.65)(0.471)}{1 - (0.65)(0.529)} = 0.529 + \frac{0.144}{0.656} = 0.749$$

   The multiple reflection factors are

$$\Gamma_{0.65} = 1.056 \qquad \Gamma_{0.85} = 1.525$$

   so there is little multiple reflection with the dark surface, but considerable multiple reflection with the bright near IR vegetated surface.