ASTR/ATOC 5560 Problem Solving Solutions Week 10

1. *An aerosol laden atmosphere has a visible optical depth of 0.4, single scattering albedo of 0.9, and asymmetry parameter of 0.75. Calculate the albedo for a solar zenith angle of* $\theta_0 = 0^\circ$ *using the delta-Eddington optically thin solution. Assume a black surface. Give the scaled optical properties.*

   The delta-scaling transformation is

   $$\tau' = (1 - \omega f)\tau \quad \omega' = \frac{(1-f)\omega}{1-\omega f} \quad g' = \frac{g-f}{1-f}$$

   For delta-Eddington the forward scattering fraction f is usually taken to be $f = g^2$, which is $f = 0.5625$ for $g = 0.75$. The delta-scaled optical properties are then

   $$\tau' = (1 - 0.9 \cdot 0.5625)(0.4) = 0.1975 \qquad \omega' = \frac{(1-0.5625)0.9}{1 - 0.9 \cdot 0.5625} = 0.7975$$

   $$g' = \frac{0.75 - 0.5625}{1 - 0.5625} = 0.4286$$

   The delta-Eddington optically thin solution for albedo is

   $$R = \left(\frac{1}{2} - \frac{3}{4} g'\mu_0\right) \omega'\tau'/\mu_0 \; = [0.5 - 0.75(0.4286)(1.0)]\,(0.7975)(0.1975) = 0.0281$$

   Note that the delta-Eddington albedo is positive whereas the regular Eddington albedo was negative $R = -0.0225$.

   Calculate the absorptance of the atmosphere.

   The Eddington absorptance is

   $$A = (1-\omega)\tau/\mu_0$$

   Using the delta scaled optical properties gives

   $$A = (1 - 0.7975)0.1975/1.0 = 0.040$$

   which is identical to the result for the unscaled properties.

   Compare the results with delta-isotropic scaling.

   Delta-isotropic is a more extreme form of delta-scaling that results in an isotropic scaled asymmetry parameter $g' = 0$. It usually give less accurate results in two-stream models, but is conceptually useful since it is one limit of delta-scaling. For delta-isotropic $f = g$, so the scaled optical properties are

   $$\tau' = (1 - 0.9 \cdot 0.75)(0.4) = 0.13 \qquad \omega' = \frac{(1-0.75)0.9}{1 - 0.9 \cdot 0.75} = 0.6923 \qquad g' = 0$$

   The delta-Eddington optically thin solution for albedo is then

   $$R = \left(\frac{1}{2} - \frac{3}{4} g'\mu_0\right) \omega'\tau'/\mu_0 \; = [0.5 - 0.75(0.0)(1.0)]\,(0.6923)(0.13) = 0.045$$