ATOC/ASTR 5560 Lab 9 Solutions

November 9, 2001

The purpose of this lab is to learn how to use a multistream polarized radiative transfer code and investigate some issues that influence the radiance field. Log in to nit and copy the following files to your directory:

`/home/rt/multistream/multi.pro`	*IDL file for the lab*
`/home/rt/multistream/readrt3output.pro`	*IDL procedure to read rt3 output file*
`/home/rt/multistream/rt3`	*executable for the doubling-adding code*
`/home/rt/multistream/runrt3`	*script to run rt3*
`/home/rt/multistream/*.f`	*Fortran source code for the RT model*

1. *Look over the* `runrt3` *script that we will use to run the polarized multistream radiative transfer model* `rt3`*. You may wish to refer to the documentation at the beginning of files* `rt3.f` *and* `radtran3.f` *for more information. The RT code reads a* layer file *and a* scattering file*, and examples of each are in the script. You will need to change only the following parameters for this lab:* `Nstokes, Nquad, Nazimuth, layerfile, deltaM, solarzenith, sfcalbedo, outfile.`

   Look at the `DOUBLING_INTEGRATION` *routine in the* `radintg3.f` *file. Compare with the doubling formulas in your notes. Note: the* `rt3` *sign convention for μ is opposite the standard convention. Count the number of matrix multiplies and inversions for a solar source problem (the number of floating point operations for these matrix operations goes as N^3 and dominates the computing time). How many of these matrix operations are done for a single layer of optical depth 1 (the initial $\delta\tau = 10^{-6}$)?*

   For the solar (exponential in τ) source and for a symmetric (homogeneous) layer there are 13 N^3 matrix operations per doubling step. The initial $\delta\tau < 10^{-6}$ so 20 doubling steps are needed ($2^{20} = 1.05 \times 10^6$) or 260 N^3 matrix operations.

2. *a) Calculate the upwelling radiance from a molecular Rayleigh scattering atmosphere with optical depth $\tau = 1$. Do two experiments: a polarized "vector" calculation with three Stokes parameters (I, Q, U) and a "scalar" calculation with only the intensity (I). Use a solar zenith angle of $60°$, zero surface albedo, and $N_{quad} = 20$. Plot the upwelling radiances in the principle plane ($\phi = 0$ and $\phi = 180°$) as a function of zenith angle using section* `PlotPolRad` *in the IDL file.*

   b) Do the same polarized vs. unpolarized experiment for the Mie scattering file from the Lab 7 mineral aerosol case. You will need to make a new layer file for the aerosol layer. Adjust the height in the layer file to give an optical depth of 1. You should use delta-M scaling and $N_{azimuth} = 20$. Plot the radiances as in a.

   Rayleigh scattering does not require delta-M scaling because it has a very smooth phase function, but the aerosol Mie phase function has a substantial forward peak, so delta-M scaling is advised. Since the highest nonzero Legendre phase function coefficient for Rayleigh scattering is $l = 2$, the highest azimuthal mode needed is $m = 2$. For the Mie phase function with 10's of terms, more azimuthal modes are needed.

c) What can you conclude about when a polarized calculation is necessary to compute an accurate intensity? Explain why by looking at the phase matrix elements P_{11} and P_{12} for the Rayleigh and Mie phase functions at $\Theta = 90^\circ$.

Clearly a fully polarized calculation is needed to compute accurate intensities for Rayleigh scattering. The error in radiances at some angles is more than 10%. However, for Mie scattering the differences between a polarized "vector" calculation and a "scalar" calculation are insignificant. Thus, with a Mie regime phase function a polarized calculation is only needed if the polarization (Stokes parameters Q, U, V) is desired, as for remote sensing polarization.

The aerosol phase matrix elements may be found from the `miegamma` phase function output. For $\Theta = 90$ they are $P_{11} = 0.240$ and $P_{12} = 0.049$. The Rayleigh phase matrix elements may be found from Rayleigh scattering theory. The Rayleigh intensity phase function is $P_{11}(\Theta) = \frac{3}{4}(1+\cos^2\Theta)$, so $P_{11} = 0.75$. For $\Theta = 90^\circ$ scattering angle, Rayleigh scattering is fully polarized with zero parallel component. Given the definition of $Q = I_{\parallel} - I_{\perp}$, the scattered Stokes parameters have $Q = -I$ so $P_{12} = -0.75$. This result may also be obtained from the Legendre series coefficients $\omega_l^{(12)}$ for $P_{12}(\Theta)$.

Therefore, for unpolarized incident light the Rayleigh scattered light at $\Theta = 90^\circ$ will be completely polarized $|Q/I| = 1$. The Mie scattered light will be only partially polarized, $|Q/I| = 0.049/0.240 = 0.2$. With a second scattering, this Q component can get converted to additional intensity I_2 , because $I_2 = P_{11}I_1 + P_{12}Q_1$. Since the maximum polarization occurs at 90° scattering angle for Rayleigh scattering, we expect that the maximum intensity difference for backward or forward scattering (more chances for two 90° scatterings) rather than side scattering.

d) Explain the differences between the Rayleigh and aerosol radiance patterns.

The upwelling radiance pattern is smoother for the Rayleigh phase function than the aerosol phase function. The Rayleigh radiance is larger in the side and backward scattering geometry. The aerosol phase function broad forward scattering peak causes the large radiances for small μ with $\phi = 0$. The Mie backscattering peak is evident near $\mu = 0.5$ at $\phi = 180$, though it is starting to wash out due to multiple scattering.

3. *a) Calculate the radiances for a layer with the $5\ \mu$m effective radius, $\lambda = 1.64\ \mu$m cloud case from Lab 8. Do two experiments: i) 0.1 km thick layer ($\tau = 1.3$) and ii) 1.0 km thick layer ($\tau = 13$). Use $N_{quad} = 40$ and $N_{azimuth} = 40$ for good angular resolution and $N_{stokes} = 1$ to make it faster. Again use a solar zenith angle of 60° and zero surface albedo. Plot the upwelling and downwelling radiances with section* `PlotRadUpDown` *in the IDL file.*

   b) Explain the angular pattern of upwelling and downwelling radiances for both optical depths. What is the maximum downwelling radiance for both cases?

   The maximum downwelling radiance for the $\tau = 1.3$ case is $8.0\ \mathrm{ster}^{-1}$, but only $0.10\ \mathrm{ster}^{-1}$ for the $\tau = 13$ case. The large downwelling radiance for the thinner case is due to the huge forward scattering peak of the cloud phase function. For an optical depth of 13 the multiple scattering completely washes out the forward scattering peak, and reduces the overall downwelling radiance substantially because so much is reflected. The optically thick transmitted radiance field is very smooth due to the multiple scattering.

As expected, the upwelling radiance is larger for $\tau = 13$ than $\tau = 1.3$. The upwelling radiance in the quadrant away from the sun ($\phi = 0$) increases for small μ for both optical depths due to the effect of the forward scattering peak.

4. *Now consider overhead sun ($\theta_0 = 0°$) for the $\tau = 13$ cloud case with a black surface.*

   a) How many azimuthal modes are needed ($N_{azimuth}$)? You can find out experimentally or theoretically.

   Only one azimuthal mode ($m = 0$, $N_{azimuth} = 0$) is needed because plane-parallel radiative transfer is azimuthally symmetric for overhead sun.

   b) What is the minimum number of quadrature angles per hemisphere needed to compute upwelling and downwelling flux *to within 0.5%. The fluxes are in the* `rt3` *output file in the rows with $\mu = +2$ (downwelling flux) and $\mu = -2$ (upwelling flux).*

   The upwelling flux from the $N_{quad} = 40$ case is $F^{\uparrow} = 0.53894$ and the downwelling flux is $F^{\downarrow} = 0.38208$. Three streams per hemisphere ($N_{quad} = 3$) gives the fluxes to within 0.3% ($F^{\uparrow} = 0.53744$ and $F^{\downarrow} = 0.38307$).

   c) Change the Lambertian surface albedo to 0.4. Using the same number of streams in part b, calculate the upwelling and downwelling flux.

   How well does the simple adding formula for the combined surface+atmosphere reflectance work in this case? Get the atmosphere R and T from part b.

   For the surface albedo of 0.4 with $N_{quad} = 3$, the fluxes are $F^{\uparrow} = 0.59824$ and $F^{\downarrow} = 0.51228$. The adding formula to combine the atmosphere and surface is

$$R_t = R_a + \frac{T_a^2 R_s}{1 - R_a R_s}$$

   The atmosphere reflectivity and transmissivity are obtained directly from the fluxes in part b, because the surface albedo was zero and the incident solar flux is 1. For $R_a = 0.537$, $T_a = 0.383$, and $R_s = 0.4$ the combined atmosphere+surface albedo is $R_t = 0.612$, which is 2.3% off from the multi-stream result of $R_t = 0.598$.

   Add the upwelling and downwelling fluxes. Is energy conserved?! Explain.

   The outgoing fluxes from the atmosphere add to $F^{\uparrow} + F^{\downarrow} = 1.11$. At first this seems to violate energy conservation becase the incident solar flux is only 1. However, this neglects the flux reflected by the surface, $F_s^{\uparrow} = (0.4)(0.512) = 0.205$, which is also incident on the atmosphere. Thus, the total incident flux is 1.20, which is greater than the outgoing flux of 1.11 because there is absorption in the layer ($\omega = 0.997$).