ATOC/ASTR 5560 Lab 7 Solutions

October 26, 2001

The purpose of this lab is to learn about the behavior of Mie scattering from particle distributions. Log in to nit and copy the following files to your directory:

`/home/rt/mie/miegamma.f`	*Fortran source for Mie code*
`/home/rt/mie/miegamma`	*Mie code executable*
`/home/rt/mie/waterindex.f`	*Fortran source for index of refraction code*
`/home/rt/mie/waterindex`	*index executable*
`/home/rt/mie/plotmie.pro`	*IDL phase function plotting file*
`/home/rt/mie/readphase.pro`	*IDL procedure to read phase function file*

1. *Look at the Mie code. Write down the name of the subroutine that performs each of the four steps of "How a Mie code works" in the class notes. Find how many times the lines inside the inner loop are executed for steps 1, 2, and 3. How will these depend on size parameter?*

	Subroutine	Process	Loopings
1	`miecalc`	Calculation of Mie coefficients a_n and b_n	$N_{radii} N_{terms}$
2	`miecross`	Calculation of the efficiencies (Q_{ext}, etc.)	$N_{radii} N_{terms}$
3	`mieangle`	Calculation of the phase function*	$N_{radii} N_{terms} N_{quad}$
4	`miedist`	Integration over the size distribution	

   N_{radii} is the number of radii in the size distribution integration, N_{terms} is the number of terms in the Mie series (number of a_n and b_n), and N_{quad} is the number of quadrature angles at with the phase function is computed.

   The number of Mie series terms is $N_{terms} \approx x$. The number of phase function angles is $N_{quad} \approx 2x$. More angles are required to represent the phase function for larger size parameters because it has more structure due to higher order $\tau_n(\Theta)$ and $\pi_n(\Theta)$ functions. Thus the number of operations for the Mie cross sections are proportional to $N_{radii} x$, while the number of operations for the Mie phase functions are proportional to $N_{radii} x^2$.

2. *The Mie code inputs gamma distribution size parameters and outputs a "scattering" file with the extinction, single scattering albedo, and Legendre series phase function and a "phase function" file with the phase function versus scattering angle. Several hundred integration steps are needed over the size distribution. As a rule of thumb, for cloud distributions the maximum radius in the integration should be about four times the effective radius. The index of refraction of water and ice are output by the* `waterindex` *program.*

   Compute Mie scattering optical properties for liquid cloud droplets with number concentration of 100 cm^{-3}, effective radius of 10 μm, and effective variance of 0.1 ($\alpha = 7$ for a gamma distribution) at wavelengths of 0.87 μm, 1.64 μm, 2.13 μm, 11.0 μm. Also calculate the scattering properties for a cloud with $N = 100\ \mathrm{cm}^{-3}$ and $r_{eff} = 5\ \mu\mathrm{m}$ at 1.64 μm. Finally, compute the Mie scattering optical properties for a mineral aerosol layer index of refraction $m = 1.56 - 0.01i$ and size distribution $N = 200\ \mathrm{cm}^{-3}$, $r_{eff} = 0.7\ \mu\mathrm{m}$, $\alpha = 1$ at 0.65 μm.

Make a table of the extinction (km^{-1}), single scattering albedo, asymmetry parameter, and number of phase function Legendre terms for the 6 cases.

The index of refractions from `waterindex` and the Mie code results are in the table below. The scattering file has Legendre coefficients for the six nonunique elements of the phase matrix, for randomly oriented particles ($P_{11}(\Theta)$, $P_{12}(\Theta)$, $P_{33}(\Theta)$, $P_{34}(\Theta)$, $P_{22} = P_{11}$, and $P_{44} = P_{33}$). The Legendre coefficients ω_l for the regular phase function $P_{11}(\Theta)$ are contained in first column after the index l. The asymmetry parameter is $g = \omega_1/3$.

λ (μm)	m	r_{eff} (μm)	N (cm^{-3})	β (km^{-1})	ω	g	N_L
0.87	$1.330 - 3.25 \times 10^{-7}i$	10	100	48.00	0.99995	0.856	510
1.64	$1.317 - 7.91 \times 10^{-5}i$	10	100	49.58	0.99404	0.843	265
1.64	$1.317 - 7.91 \times 10^{-5}i$	5	100	13.07	0.99705	0.800	130
2.13	$1.296 - 3.96 \times 10^{-4}i$	10	100	50.51	0.9785	0.843	203
11.00	$1.155 - 9.82 \times 10^{-2}i$	10	100	38.87	0.4748	0.924	39
0.65	$1.56 - 0.01i$	0.7	200	0.321	0.8880	0.682	75

3. *Compare the Mie extinction with the simple formula assuming geometric optics for the extinction in terms of the liquid water content and effective radius. Compare the single scattering albedo with the weakly absorbing geometric optics formula:*

$$1 - \omega \approx -\frac{8\pi m_i}{3\lambda_{in}} r_{eff} ,$$

where $\lambda_{in} = \lambda/m_r$ is the wavelength inside the particle.

Explain why the simple formulas work in some cases and not in others.

The geometric optics formula for extinction in terms of liquid water content W and effective radius r_e is

$$\beta = \frac{3W}{2\rho_l r_e}$$

From the notes, the liquid water content W is related to the gamma distribution parameters, N, $b = (\alpha + 3)/r_e$, and α by

$$W = \frac{4\pi\rho_l}{3} N b^{-3} (\alpha + 3)(\alpha + 2)(\alpha + 1) \ = \frac{4\pi\rho_l}{3} \frac{(\alpha + 2)(\alpha + 1)}{(\alpha + 3)^2} r_e^3 N$$

For the first cloud, $N = 100\ \mathrm{cm}^{-1}$, $r_{eff} = 10\ \mu$m, $\alpha = 7$ so the liquid water content is

$$W = \frac{4\pi(10^6\ \mathrm{g/m}^3)}{3} \frac{72}{100} (10^{-5}\ \mathrm{m})^3 (100 \times 10^6\ \mathrm{m}^{-3} \ = 0.302\ \mathrm{g/m}^3$$

The geometric optics limit extinction can be found directly in terms of number concentration N by

$$\beta = 2\pi r_e^2 N \frac{(\alpha + 2)(\alpha + 1)}{(\alpha + 3)^2}$$

$$\beta = 4.524 r_e^2 N \; = 4.524 (10\ \mu\text{m})^2 (100\ \text{cm}^{-3})(10^{-4}\ \text{cm}/\mu\text{m})^2 (10^5\ \text{cm/km}) = 45.24\ \text{km}^{-1}$$

This equation is used in a small program to create the table below:

λ (μm)	r_{eff} (μm)	β_{Mie} (km^{-1})	β_{GO} (km^{-1})	ω_{Mie}	ω_{GO}
0.87	10.0	48.00	45.24	0.99995	0.99996
1.64	10.0	49.58	45.24	0.99404	0.99468
1.64	5.0	13.07	11.31	0.99705	0.99734
2.13	10.0	50.51	45.24	0.97850	0.97981
11.00	10.0	38.87	45.24	0.47480	0.13619
0.65	0.7	0.32	0.23	0.88800	0.85926

Even though only the $\lambda = 0.87$ μm case is actually close to the geometric optics limit, the simple formulas for the extinction and single scattering albedo work quite well for the other solar wavelength cloud cases. The formulas do not work well for the aerosol case because the size parameter is much smaller than valid for geometric optics (the Mie regime resonant oscillation causes the actual extinction to be larger). The $\lambda = 11$ μm case is both not in the geometric optics limit and not weakly absorbing, so it fails especially badly for the single scattering albedo.

4. *Graph the phase function (P_{11}) for the six cases. Use two plots of three each.*

   Make a table of the effective size parameter x_{eff}, the peak phase function $P(0)$, and the approximate scattering angle at half the phase function peak $\Theta_{1/2}$. Is the actual functional relation between these three quantities as expected?

   The three cloud phase functions on the first page are very similar, except the diffraction peak narrows and the rainbow and glory features become more distinct for the larger size parameters. (The small oscillations in the $\lambda = 0.87$ μm case are due to not having enough terms in the size distribution integration). The phase function for the $\lambda = 11$ μm cloud case shows very little scattering backwards due to absorption within the droplets. The mineral aerosol phase function has a lower, wider forward peak due to the small size parameter but a substantial backscattering peak from the high real part of the index of refraction.

   See the table below. The effective size parameter is $x_{eff} = 2\pi r_{eff}/\lambda$. The half power scattering angle is estimated from the phase function file.

λ (μm)	r_{eff} (μm)	x_{eff}	$P(0)$	$\Theta_{1/2}$ (degrees)	$\Theta_{1/2} x_{eff}$ (degrees)	$P(0)/x_{eff}^2$
0.87	10	72.2	2956	1.1	78	0.57
1.64	10	38.3	865	2.0	77	0.59
1.64	5	19.2	232	4.0	77	0.63
2.13	10	29.5	532	2.6	77	0.61
11.00	10	5.7	47.0	12.9	74	1.45
0.65	0.7	6.8	38.9	9.4	64	0.84

From diffraction theory, which applies to the geometric optics limit, we expect an inverse relation between the width of the forward scattering peak and the size parameter. The peak of the phase function should goes as x^2, since the phase function is normalized to unity and the solid angle goes as $\Theta_{1/2}^2$. The last two columns of the table show that these relationships hold for the solar wavelength cloud cases. From the diffraction pattern plot, $x \sin(\Theta_{1/2}) = 1.6$ or $\Theta_{1/2} = 57.3(1.6/x) = 92°/x$, but that was for a single sphere, rather than a distribution specified by the effective radius.

Explain why the asymmetry parameter is highest for the $\lambda = 11$ μm cloud case even though the size parameter is lower than for the shorter wavelength cloud cases.

Light that penetrates significantly into the particle is absorbed, so most of the scattering is due to diffraction, which is around the forward direction.

5. *Write down the 4×4 Stokes phase matrix for the $\lambda = 0.87$ μm cloud case for a scattering angle of $125°$. If unpolarized light is incident on these cloud droplets, what is the resulting Stokes vector normalized by the scattered intensity? What is the degree of linear polarization of the scattered light?*

   The phase matrix at $125.2°$ is

$$\begin{bmatrix} 0.0487 & -0.0224 & 0 & 0 \\ -0.0224 & 0.0487 & 0 & 0 \\ 0 & 0 & 0.00486 & 0.0239 \\ 0 & 0 & -0.0239 & 0.00486 \end{bmatrix}$$

   Unpolarized light is of the form $[I, Q, U, V] = [1, 0, 0, 0]$. Light scattering is represented by multiplying the incident Stokes vector by the phase matrix. The resulting normalized scattered Stokes vector is $[1, -0.46, 0, 0]$. The degree of linear polarization is $\sqrt{Q^2 + U^2}/I = 0.46$.