**Radiative Processes in Planetary Atmospheres — Homework 5**
Due: November 14, 2001

Please show your work.

1. a) Show that for an optically thin atmosphere and a very dark surface, the combined atmosphere-surface reflectivity is simply the sum of the reflectivity of the atmosphere and surface.

   b) In visible and near IR remote sensing the radiance is often normalized by the incident solar flux to give the reflectance defined by $R = \pi I/(\mu_0 S_0)$. Calculate this reflectance in the direction $\theta = 30°, \phi = 140°$ from an atmosphere containing only a 1.0 km layer of the mineral aerosol from Lab 8. The surface is Lambertian with an albedo of 0.04. The direction to the sun is $\theta_0 = 30°, \phi_0 = 180°$. Calculate the total reflectance for two cases i) the actual aerosol Mie phase function, and ii) the Henyey-Greenstein phase function for the same asymmetry parameter.

2. a) Calculate the delta-isotropic scaled optical properties ($\beta, \omega, g$) for the $\lambda = 11$ $\mu$m cloud case in Lab 7.

   b) Given the results in part a, what two approximate solutions to the thermally emitting and scattering radiative transfer equation would be both accurate and computationally efficient? Explain why. Hint: the second approximation is more accurate than the first.

3. a) The similiarity parameter in radiative transfer is defined as $s = \sqrt{(1-\omega)/(1-\omega g)}$, where $\omega$ is the single scattering albedo and $g$ is the asymmetry parameter. Show that the similiarity parameter is unchanged in delta scaling.

   b) Argue why the semi-infinite limit ($\tau \rightarrow \infty$) for the albedo of a homogeneous atmosphere should depend only on the similiarity parameter $s$ and the sun angle $\mu_0$, rather than on all three parameters, $\omega$, $g$, and $\mu_0$, separately.

   c) By setting $g = 0$, derive a simplified expression for the semi-infinite albedo in the Eddington approximation in terms of $s$ and $\mu_0$. Start with the Meador and Weavor semi-infinite solution (eq 22).

   d) Describe how the semi-infinite albedo $R$ depends on $\omega$ and $g$ using the expression for the similiarity parameter (a graph of $R$ versus $s$ might be useful here).

4. This question is about the adding-doubling method in the two-stream approximation. The azimuthally symmetric discrete ordinate solar radiative transfer equation

is obtained by using one quadrature angle $\mu_1$ per hemisphere and two terms in the Legendre phase function series. For upwelling radiance $I^+$ the two-stream RTE is

$$\mu_1 \frac{dI^+}{d\tau} = I^+ - \frac{\omega}{2}\left[(1 + 3g\mu_1^2)I^+ + (1 - 3g\mu_1^2)I^-\right] - \frac{\omega}{4\pi}(1 - 3g\mu_1\mu_0)S_0 e^{-\tau/\mu_0}$$

where $I^\pm = I(\tau, \pm\mu_1)$ are the discrete ordinate radiances. For downwelling radiance $I^-$ the two-stream RTE is

$$\mu_1 \frac{dI^-}{d\tau} = -I^- + \frac{\omega}{2}\left[(1 - 3g\mu_1^2)I^+ + (1 + 3g\mu_1^2)I^-\right] + \frac{\omega}{4\pi}(1 + 3g\mu_1\mu_0)S_0 e^{-\tau/\mu_0}$$

a) Calculate the reflection $R$, transmission ($T$), and source ($S^\pm$) coefficients for an infinitesimally thin layer of optical depth $\delta\tau$. Note: for a homogeneous layer $R^+ = R^-$ and $T^+ = T^-$.

b) If we do the standard direct/diffuse solar radiation separation, what are the boundary conditions (i.e. incident radiances) on the layer in the interaction principle. Hence what terms in the interaction principle gives the radiance outgoing from the atmosphere.

c) Use the doubling relations to compute the albedo as a function of optical depth for $\omega = 1.0$ and $\omega = 0.99$. Use a solar senith angle of 30° ($\mu_0 = 0.866$), asymmetry parameter $g = 0.85$, and a black surface. The double gaussian quadrature angle is $\mu_1 = 1/2$ with weight $w = 1$. Use delta scaling with $f = g^2$. Start with $\delta\tau = 1/1024$ and double to $\tau = 64$. Convert the discrete ordinate radiance to albedo and plot the two albedo curves. Hand in the program you use to do the computations.