

Radiative Processes in Planetary Atmospheres — Homework 4

Due: October 29, 2001

Please show your work.

- Derive an expression relating the Rayleigh scattering volume absorption coefficient for a particle size distribution to the integrated volume distribution (V , the volume of particles per volume of air).
 - What is the volume fraction for a log-normal size distribution with $N_{tot} = 100 \text{ cm}^{-3}$, $r_0 = 0.2 \text{ }\mu\text{m}$, and $\sigma = 0.40$? Calculate the volume absorption coefficient for ammonium sulfate at a wavelength of $10 \text{ }\mu\text{m}$ ($m = 2.190 - 0.130i$). Compare with your results in lab 6.
- Rayleigh scattering for spherical particles is a limiting case of Mie scattering as the size parameter $x \rightarrow 0$. In this limit the scattered field coefficients a_n and b_n are all negligible except for a_1 which is

$$a_1 = \frac{2i m^2 - 1}{3 m^2 + 2} x^3,$$

using the notation of Liou (1980) or Bohren and Huffman (1983).

Use the Mie theory results to derive the following quantities for Rayleigh scattering:

- the scattering efficiency Q_{sca} and the extinction efficiency Q_{ext} ,
- the scattering amplitudes $S_1(\Theta)$ and $S_2(\Theta)$,
- the phase function for intensity $P_{11}(\Theta)$.

Note: the Mie angular functions are defined in terms of associated Legendre polynomials and the relevant one is $P_1^1(\cos \Theta) = \sin \Theta$. Also, $\text{Re}[iz] = -\text{Im}[z]$, $|z|^2 = (\text{Re}[z])^2 + (\text{Im}[z])^2$.

- Consider the case of a nonabsorbing sphere in this limit. What is the extinction efficiency derived above in this case? Is this result physical (compare with the scattering)? What might be the cause of this dilemma? In this limit of Mie theory what process does Q_{ext} really measure?
- The parameters needed for a two-stream radiative transfer flux calculation in a scattering atmosphere are the optical depth τ , the single scattering albedo ω , and the asymmetry parameter g . Consider the cloud and haze layer from about 50 to 65 km in the Venusian atmosphere. The cloud and haze droplets are sulfuric acid. At a wavelength of 550 nm the index of refraction is $m = 1.45$. Say the optical depth of the cloud droplets is 25 and of the haze is 3. The asymmetry parameter of the cloud droplets is 0.78 and of the haze particles is 0.70.

- a) Write down an expression for the total optical properties of the layer τ , ω , and g from these optical properties for the cloud and haze particle scattering.
- b) Compute the total τ , ω , and g for the layer.
4. You see the full moon through a uniform thin cloud and notice a well defined disk of light around the moon. The outer part of the disk has a brownish-red tinge. The diameter of the disk is 10 times that of the moon. What is the disk called? Why is it colored? What is the approximate radius of particles in the cloud? Comment on the width of the particle size distribution. What type of cloud is this?