**ATOC/ASTR 5560 Homework 1 Solutions**
Due: September 17, 2001

1. *Assume the Sun radiates as a blackbody at 5783 K and is a uniform sphere with radius* $6.96 \times 10^5$ *km. Calculate the broadband radiance* **and** *irradiance of sunlight at the orbits of Venus and Earth.*

   Since we want broadband flux we can use the Stefan-Boltzmann Law to find the flux at the surface of the Sun:

$$F_\odot = \sigma T_e^4/\pi = (5.67 \times 10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}})(5783\ \mathrm{K})^4 = 6.34 \times 10^7\ \mathrm{W/m^2}$$

   The solar flux at the orbit of the Earth is obtained with the inverse square law. The inverse square law is derived by setting the solar power radiated through a sphere with 1 AU radius to that emitted over the whole surface of the Sun.

$$F_\oplus = F_\odot \left(\frac{R_\odot}{r_\oplus}\right)^2 = (6.34 \times 10^7\ \mathrm{W/m^2}) \left(\frac{6.96 \times 10^5\ \mathrm{km}}{1.50 \times 10^8\ \mathrm{km}}\right)^2 = 1365\ \mathrm{W/m^2}$$

   The solar flux at Venus is

$$F = (6.34 \times 10^7\ \mathrm{W/m^2}) \left(\frac{6.96 \times 10^5\ \mathrm{km}}{1.08 \times 10^8\ \mathrm{km}}\right)^2 = 2634\ \mathrm{W/m^2}$$

   There are two ways to get the radiance of the Sun. One is to remember that in a transparent medium the intensity does not change with distance along a ray. Therefore we just need to find the radiance at the surface of the Sun. The assumed isotropic blackbody flux is converted to radiance by dividing by $\pi$:

$$I = \sigma T_e^4/\pi = 2.02 \times 10^7 \mathrm{W\ m^{-2}\ sr^{-1}}$$

   The second method is to calculate the radiance from the flux at the Earth's orbit using the solid angle of the Sun. The solid angle of the Sun is

$$\Omega = \frac{A_{sun}}{r_\oplus^2} = \frac{\pi R_\odot^2}{r_\oplus^2} = \pi \left(\frac{6.96 \times 10^5\ \mathrm{km}}{1.50 \times 10^8\ \mathrm{km}}\right)^2 = 6.76 \times 10^{-5}\ \mathrm{sr^{-1}}$$

$$I = \frac{F_\oplus}{\Omega} = \frac{1365\ \mathrm{W/m^2}}{6.76 \times 10^{-5}\ \mathrm{sr^{-1}}} = 2.02 \times 10^7 \mathrm{W\ m^{-2}\ sr^{-1}}$$

2. *a) Ozone amount is measured in Dobson units. A Dobson unit is a milli-atmospheric-cm, where an atm-cm is the thickness of the gas if reduced to standard temperature (273 K) and pressure (101.3 kPa). Find the ozone amount over Boulder (latitude 40.0, longitude -105.0)*

*on September 5, 2001 using the TOMS Web site (*`http://jwocky.gsfc.nasa.gov/teacher/ozone_overhead.html`*).*

*Convert the total ozone in Dobson units to absorber amount in g/cm*$^2$*. Hint: a gas at standard temperature and pressure occupies* $2.241 \times 10^4\ \mathrm{cm^3/mol}$.

The TOMS Web site reports 277 Dobson units.

$$u = \Delta z_{STP}\ \rho_{O_3,STP} = \Delta z \frac{M}{V}\ = \frac{(0.277\ \mathrm{cm})(48\ \mathrm{g/mol})}{2.241 \times 10^4\ \mathrm{cm^3/mol}} = 5.93 \times 10^{-4}\ \mathrm{g/cm^2}$$

*b) Water vapor amount is measured in precipitable cm or inches, which is the height of the column of water resulting from condensing all of the water vapor out. At 12 GMT on September 5, 2001 the Denver radiosonde recorded 0.56 inches. Convert this to absorber amount in g/cm*$^2$*.*

$$u = (0.56\ \mathrm{in})(2.54\ \mathrm{cm/in})(1.00\ \mathrm{g/cm^3}) = 1.42\ \mathrm{g/cm^2}$$

*c) Carbon dioxide concentration is measured in parts per million by volume. There are small seasonal and geographic variations and an increasing trend, but the current value is now about 370 ppmv (as measured by the Climate Monitoring and Diagnostic Laboratory at their four baseline observatories; see* `http://www.cmdl.noaa.gov/ccgg/insitu/index.html`*). Convert this concentration to absorber amount in g/cm*$^2$ *above the Mauna Loa observatory (3400 m altitude, 680 mb pressure).*

$$u = \frac{q_a}{g} p_s\ = \frac{(3.70 \times 10^{-4})(44.0/29.0)(6.80 \times 10^4\ \mathrm{N/m^2})}{9.8\ \mathrm{m/s^2}} = 3.90\ \mathrm{kg/m^2} = 0.390\ \mathrm{g/cm^2}$$

3. *A hypothetical Mars lander probe carries a narrow field of view tracking sunphotometer operating at* $\lambda = 0.5\ \mu m$ *wavelength. One morning the instrument measures the following voltages as Sun rises above the horizon.*

| *Solar Zenith Angle* | *Voltage* |
|---|---|
| 75° | *1.534* |
| 70° | *1.937* |
| 60° | *2.435* |

*a) What is the aerosol optical depth at 0.5* $\mu m$*?*

Assuming the sun photometer detector is linear and the atmosphere is uniform during the measurement, the voltage response of the sun photometer follows Beer's Law:

$$V = V_0 \exp(-\tau m)$$

where $\tau$ is the atmospheric optical depth and $m$ is the relative air mass. For solar zenith angles less than $80^\circ$ the air mass can be obtained from $m = 1/\cos\theta_0$.

The Langley plot technique can be used to obtain atmospheric optical depth. This is a plot of log voltage (or transmission or radiance) measured by the sunphotometer as a function of air mass. Taking the log of Beer's Law shows that the slope is the negative of optical depth.

$$\ln V = \ln V_0 - \tau m$$

After verifying that the Langley plot is a straight line, the slope may be found from two points, giving

$$\tau = -\frac{\ln V_1 - \ln V_2}{m_1 - m_2} = -\frac{\ln 1.534 - \ln 2.435}{3.864 - 2.00} = 0.248$$

This is the total optical depth. Since the Martian atmosphere is so thin and mainly consists of $CO_2$, there are no other absorbers at this wavelength. Therefore the aerosol optical depth is 0.248.

*b) What assumptions about the atmosphere did you have to make to derive the aerosol optical depth?*

The Langley plot technique assumes that the atmosphere is horizontally uniform and is not changing during the measurement. If the optical depth changes then the slope is not constant and so the Langley plot will not give a straight line. The Langley method also assumes that the spectral band is narrow enough that Beer's law applies to the transmission. Finally, you had to assume that there were no other sources of extinction other than aerosols at this wavelength.

4. *The Atmospheric Radiation Measurement program deploys a number of Microwave Water Radiometers (MWR) at its three sites around the world. The MWR measures the zenith viewing brightness temperature at 23.8 and 31.4 GHz looking up from the ground. These two brightness temperatures are inverted to obtain the integrated water vapor and cloud liquid water mass (see http://www.arm.gov/ for more on the MWR). Assume the sky is known to be clear from lidar data, in which case the 23.8 GHz brightness temperature can be related to the integrated water vapor amount.*

   *a) Derive an equation for the integrated water vapor amount (g/cm$^2$) from the brightness temperature assuming the mean atmospheric radiating temperature $T_{mr}$ and the mass absorption coefficient $k_\nu$ are known. Include the cosmic background, which is blackbody radiation at $T_{cb} = 2.7$ K. Use the Rayleigh-Jeans approximation, i.e. use brightness temperatures for all radiances.*

   Let $\tau_n u$ be the optical depth at 23.8 GHz, and $k_\nu$ be the mass absorption coefficient. Then $\tau_\nu = k_\nu u$ where $u$ is the water vapor amount.

   Using the Rayleigh-Jean limit the single layer thermal radiative transfer equation for downwelling radiance at $\mu = -1$:

$$T_b = e^{-\tau_\nu} T_{cb} + (1 - e^{-\tau_\nu}) T_{mr}$$

Solve this for the transmission factor to get

$$e^{-\tau_\nu} = \frac{T_{mr} - T_b}{T_{mr} - T_{cb}}$$

Then solve for $u$ to obtain

$$u = -\frac{1}{k_\nu} \ln \left[ \frac{T_{mr} - T_b}{T_{mr} - T_{cb}} \right]$$

*b) Calculate the water vapor amount corresponding to a 23.8 GHz brightness temperature of 64.3 K if the mean radiating temperature is $T_{mr}$=288 K and the mass absorption coefficient is $k_\nu$=0.058 cm$^2$/g. These values are for a tropical atmosphere.*

The water vapor amount is

$$u = -\frac{1}{0.058 \text{ cm}^2/\text{g}} \ln \left[ \frac{288 \text{ K} - 64.3 \text{ K}}{288 \text{ K} - 2.7 \text{ K}} \right]$$

$$e^{-\tau_n u} = 0.7841 \quad \tau_\nu = 0.2432 \quad u = 4.194 \text{ g/cm}^2$$

*c) Determine numerically the uncertainty in the water vapor amount from a 0.5 K brightness temperature uncertainty and, separately, from a 5 K mean radiating temperature uncertainty.*

For the observed brightness temperature uncertainty add 0.5 K to the original value and derive a new water vapor amount

$$T_b = 64.8 \text{ K} \quad u = 4.232 \text{ g/cm}^2 \quad \Delta u = 0.038 \text{ g/cm}^2 \quad 0.9\% \text{ error}$$

For the uncertainty in the mean radiating temperature:

$$T_{mr} = 293 \text{ K} \quad u = 4.112 \text{ g/cm}^2 \quad \Delta u = -0.082 \text{ g/cm}^2 \quad 2.0\% \text{ error}$$