

Examples Week 8

1. Using the fundamental extinction formula calculate the extinction cross section for spheres in the Rayleigh limit. Derive the scattering amplitudes from the equations given for the electric field in the far field in Mie theory and the scattered electric field in the Rayleigh scattering derivation. Compare to the Rayleigh absorption cross section.

The fundamental extinction formula is given by

$$C_{ext} = \frac{4\pi}{k^2} \text{Re}[S_{1,2}(0^\circ)]$$

In Mie theory, the electric field in the far field is given by

$$\begin{bmatrix} E_{\parallel} \\ E_{\perp} \end{bmatrix}_{sca} = \frac{\exp(-ikR + ikz)}{ikR} \begin{bmatrix} S_2 & S_3 \\ S_4 & S_1 \end{bmatrix} \begin{bmatrix} E_{\parallel}^0 \\ E_{\perp}^0 \end{bmatrix}$$

Since $S_3 = S_4 = 0$ for spheres, this gives

$$E_{\parallel,sca} = \frac{\exp(-ikR + ikz)}{ikR} S_2 E_{\parallel}^0$$

$$E_{\perp,sca} = \frac{\exp(-ikR + ikz)}{ikR} S_1 E_{\perp}^0$$

From the derivation of Rayleigh scattering, we have

$$E_{\perp,sca} = E_{\perp,0} \frac{\exp -k(R - ct)}{R} k^2 \alpha \quad E_{\parallel,sca} = E_{\parallel,0} \frac{\exp -k(R - ct)}{R} k^2 \alpha \cos \Theta$$

Combining these, we find that

$$S_1(\Theta) = ik^3 \alpha \quad S_2(\Theta) = ik^3 \alpha \cos \Theta$$

$$S_1(0^\circ) = S_2(0^\circ) = ik^3 \alpha = ix^3 \left\{ \frac{m^2 - 1}{m^2 + 2} \right\}$$

Therefore,

$$C_{ext} = \frac{4\pi}{k^2} \text{Re} \left\{ ix^3 \left[\frac{m^2 - 1}{m^2 + 2} \right] \right\} = -4\pi kr^3 \text{Im} \left\{ \frac{m^2 - 1}{m^2 + 2} \right\}$$

The Rayleigh absorption cross section is given by

$$C_{abs} = -\frac{8\pi^2 r^3}{\lambda} \text{Im} \left\{ \frac{m^2 - 1}{m^2 + 2} \right\}$$