Examples Week 7

1. Determine the optical properties of a atmosphere layer with molecular water vapor absorption and aerosol scattering. The water vapor density is $\rho_v = 10 \text{ g/m}^3$ and mass absorption coefficient is $k_\lambda = 0.03 \text{ m}^2\text{/kg}$. The aerosol has a volume extinction coefficient of $\beta_{ext} = 0.2 \text{ km}^{-1}$, single scattering albedo of $\omega = 0.9$, and asymmetry parameter of $g = 0.75$.

We want to find β, ω, and g for the combination of water vapor and aerosol. First, calculate the volume extinction coefficient due to water vapor

$$\beta_{H2O} = k_\lambda \rho_v = (0.03 \text{ m}^2\text{/kg})(0.010 \text{ kg/m}^3) = 0.0003 \text{ m}^{-1} = 0.3 \text{ km}^{-1}$$

Since molecular absorption does not scatter light, the single scattering albedo is zero ($\omega_{H2O} = 0$).
The total extinction is the sum of the two extinction coefficients:

$$\beta_{tot} = \beta_{H2O} + \beta_{aero} = 0.5 \text{ km}^{-1}$$

The total single scattering albedo is found by adding the scattering coefficients:

$$\beta_{tot}\omega_{tot} = \beta_{H2O}\omega_{H2O} + \beta_{aero}\omega_{aero}$$

$$\omega_{tot} = \frac{\beta_{H2O}\omega_{H2O} + \beta_{aero}\omega_{aero}}{\beta_{tot}} = \frac{(0.2 \text{ km}^{-1})(0.9)}{0.5 \text{ km}^{-1}} = 0.36$$

The total asymmmetry parameter is found by adding the scattering coefficient weighted asymmetry parameters:

$$\beta_{tot}\omega_{tot}g_{tot} = \beta_{H2O}\omega_{H2O}g_{H2O} + \beta_{aero}\omega_{aero}g_{aero}$$

$$g_{tot} = \frac{\beta_{aero}\omega_{aero}g_{aero}}{\beta_{tot}\omega_{tot}} = \frac{\beta_{aero}\omega_{aero}g_{aero}}{\beta_{aero}\omega_{aero}} = g_{aero} = 0.75$$

2. Calculate the Stokes parameters (I, Q, U, V) for the E field represented in Fig. (c) on page 10 of this week's notes. Identify the type of polarization and the degree of polarization.

For case (c) the relation between the two components of the electric field vector are equal amplitudes ($\mathcal{E}_\perp = \mathcal{E}_\parallel$) and phases shifted by 90° ($\Phi_\perp = \Phi_\parallel + \frac{\pi}{2}$).

At a given location the two components of the E field vector for this single wave are

$$E_\parallel = \mathcal{E}_\parallel e^{-ikct} \qquad E_\perp = \mathcal{E}_\perp e^{-ikct+i\pi/2}$$

The Stokes vectors are defined by

$$I = \langle |E_\parallel|^2 \rangle + \langle |E_\perp|^2 \rangle \qquad Q = \langle |E_\parallel|^2 \rangle - \langle |E_\perp|^2 \rangle$$

$$U = \langle E_\parallel E_\perp^* + E_\perp E_\parallel^* \rangle \qquad V = i\langle E_\parallel E_\perp^* - E_\perp E_\parallel^* \rangle$$

Therefore, the Stokes parameters are

$$I = \mathcal{E}_\parallel^2 |e^{-ikct}|^2 + \mathcal{E}_\perp^2 |e^{-ikct+i\pi/2}|^2 \; = 2\mathcal{E}^2$$

$$Q = \mathcal{E}_\parallel^2 - \mathcal{E}_\perp^2 \; = 0$$

$$U = \langle \mathcal{E}_\parallel e^{-ikct} \mathcal{E}_\perp e^{+ikct-i\pi/2} + \mathcal{E}_\parallel e^{ikct} \mathcal{E}_\perp e^{-ikct+i\pi/2} \rangle$$

$$U = \mathcal{E}^2 \left[e^{-i\pi/2} + e^{+i\pi/2} \right] = \mathcal{E}^2 [-i + i] = 0$$

$$V = i\langle \mathcal{E}_\parallel e^{-ikct} \mathcal{E}_\perp e^{+ikct-i\pi/2} - \mathcal{E}_\parallel e^{ikct} \mathcal{E}_\perp e^{-ikct+i\pi/2} \rangle$$

$$V = i\mathcal{E}^2 \left[e^{-i\pi/2} - e^{+i\pi/2} \right] = i\mathcal{E}^2 [-i - i] = 2\mathcal{E}^2 = I$$

This is circularly polarized light, since $|V| = I$. The sign of V determines whether it is left or right circularly polarized, but both sign conventions are used. Since this is a single wave it must be fully polarized and the degree of polarization $\sqrt{Q^2 + U^2 + V^2}/I = 1$. The degree of linear polarization is zero since both Q and U are zero.