Examples Week 6

1. Calculate the upwelling radiance at $\mu = 0.6$ for the spectral band 980-1100 cm^{-1} in a subarctic winter atmosphere. This spectral region is band 12 in Fu's k-distribution, which is the 9.6 μm ozone band.

Treat the ozone layer as an isothermal layer with temperature of 217 K. The ozone absorber amount is 0.010 kg/m^2. Assume a black surface at a temperature of 257 K. Water vapor absorption can be ignored.

The k-distribution parameters are

j	Δg_j	k_j (m^2/kg)
1	0.45	4.2
2	0.30	41.6
3	0.20	238
4	0.04	1299
5	0.01	4073

We can use the single layer form of the thermal radiative transfer equation, since we have an isothermal layer:

$$I_\nu(\infty, \mu) = B_\nu(T_s)e^{-\tau_\nu/\mu} + B_\nu(T_a)(1 - e^{-\tau_\nu/\mu})$$

where $I_\nu(\infty, \mu)$ is the upwelling radiance at the top of the atmosphere, τ_ν is the monochromatic optical depth, T_a is the absorber weighted atmosphere temperature, and T_s is the temperature of the black surface.

To use this with a k-distribution to find the band integrated radiance, we need the band integrated Planck function. We can approximate this by

$$B_{\Delta\nu}(T) = \int_{\nu_1}^{\nu_2} B_\nu(T)d\nu \ = \ (\nu_2 - \nu_1)\ B_{\bar{\nu}}(T)$$

Evaluating the Planck function for the two temperatuers at $\bar{\nu} = 1040\ \mathrm{cm}^{-1}$ gives

$$B_{\Delta\nu}(T_s) = (0.03978\ \mathrm{W\ m^{-2}sr^{-1}cm})(120\ \mathrm{cm}^{-1}) = 4.774\ \mathrm{W\ m^{-2}sr^{-1}}$$

$$B_{\Delta\nu}(T_a) = (0.01358\ \mathrm{W\ m^{-2}sr^{-1}cm})(120\ \mathrm{cm}^{-1}) = 1.629\ \mathrm{W\ m^{-2}sr^{-1}}$$

Using the k-distribution the upwelling band integrated radiance is

$$I_{\Delta\nu}(\infty, \mu) = \sum_{j=1}^{5} \Delta g_j\ \left[B_{\Delta\nu}(T_s)e^{-k_j u_{O3}/\mu} + B_{\Delta\nu}(T_a)(1 - e^{-k_j u_{O3}/\mu})\right]$$

The optical path and upwelling radiance for each “k” are listed below:

j	Δg_j	k_j (m^2/kg)	$k_j u/\mu$	I_j (W m^{-2}sr^{-1})
1	0.45	4.2	0.070	4.561
2	0.30	41.6	0.693	3.201
3	0.20	238	3.967	1.689
4	0.04	1299	21.65	1.629
5	0.01	4073	67.88	1.629

$$I_{\Delta\nu}(\infty, \mu) = 0.45(4.561) + 0.30(3.201) + 0.20(1.689) + 0.04(1.629) + 0.01(1.629)$$

$$I_{\Delta\nu}(\infty, \mu) = 3.432\ \mathrm{W\ m^{-2}sr^{-1}}$$

If we instead used the band mean optical path ($\langle\tau/\mu\rangle = 2.578$) in the monochromatic radiative transfer equation, the band integrated radiance would have been $I_{\Delta\nu}(\infty, \mu) = 1.868\ \mathrm{W\ m^{-2}sr^{-1}}$. Thus, the effect of the spectrally varying ozone absorption is to dramatically increase the band mean transmission over the spectrally gray case.