Examples Week 5

2. Calculate the band mean transmission using the Malkmus random band model for H_2O from 400 to 500 cm^{-1}. The layer is from 6 to 7 km in a tropical atmosphere where the mean temperature is 260 K and the pressure is 462 mb. The absorber amount of water vapor is 2.16×10^{21} molecules/cm^2. Calculate the mean transmission for this layer and with 1/10 and 10 times the water vapor. The line parameters from table 17.5 in Lenoble for 260 K and 1013 mb are $\sum_i S_i = 2.69 \times 10^{-19}$ cm/molecule and $\sum_i \sqrt{S_i \alpha_i} = 7.68 \times 10^{-10}\ \mathrm{molecule}^{-1/2}$.

The line parameters are for the correct temperature, but not for the desired pressure. The halfwidth of each absorption line α_i scales linearly with the pressure, so we can adjust the parameter accordingly:

$$\sum_i \sqrt{S_i \alpha_i} = \sum_i \sqrt{S_i \alpha_{0,i} \frac{p}{p_0}}$$

$$\sum_i \sqrt{S_i \alpha_i} = 7.68 \times 10^{-10}\ \mathrm{molec}^{-1/2} \sqrt{462/1013} = 5.187 \times 10^{-10}\ \mathrm{molec}^{-1/2}$$

We need to find the parameters in the Malkmus random band model for mean transmission:

$$\bar{\mathcal{T}}(u) = \exp\left[-\frac{\pi\bar{\alpha}}{2\delta}\left(\left[1 + \frac{4\bar{S}u}{\pi\bar{\alpha}}\right]^{1/2} - 1\right)\right]$$

Using the weak and strong limits of the band model, the band model parameters can be related to the absorption line parameters in the band:

$$\frac{\bar{S}}{\delta} = \frac{1}{\Delta\nu}\sum_{i=1}^{n} S_i \qquad \frac{\sqrt{\pi\bar{\alpha}\bar{S}}}{\delta} = \frac{2}{\Delta\nu}\sum_{i=1}^{n}\sqrt{S_i\alpha_i}$$

For the Malkmus model we need the combination

$$\frac{4\bar{S}}{\bar{\alpha}\pi} = 4\left(\frac{\bar{S}/\delta}{\sqrt{\pi\bar{\alpha}\bar{S}}/\delta}\right)^2 = \left(\frac{\sum_i S_i}{\sum_i \sqrt{S_i\alpha_i}}\right)^2$$

$$\frac{4\bar{S}}{\bar{\alpha}\pi} = \left(\frac{2.69 \times 10^{-19}\ \mathrm{cm/molec}}{5.187 \times 10^{-10}\ \mathrm{molec}^{-1/2}}\right)^2 = 2.690 \times 10^{-19}\ \mathrm{cm^2/molec}$$

and we also need

$$\frac{\pi\bar{\alpha}}{2\delta} = \frac{\left(\sqrt{\pi\bar{\alpha}\bar{S}}/\delta\right)^2}{2\bar{S}/\delta} = \frac{2}{\Delta\nu}\frac{\left(\sum_i \sqrt{S_i\alpha_i}\right)^2}{\sum_i S_i}$$

$$\frac{\pi\bar{\alpha}}{\delta} = \frac{2}{100\ \mathrm{cm}^{-1}}\frac{(5.187\times 10^{-10}\ \mathrm{molec}^{-1/2})^2}{2.69\times 10^{-19}\ \mathrm{cm/molec}} = 0.0200$$

The mean transmission of the 6 to 7 km layer of H_2O from 400 to 500 cm^{-1} is

$$\bar{\mathcal{T}}(u) = \exp\left[-0.020\left(\left[1 + (2.690\times 10^{-19}\ \mathrm{cm}^2/\mathrm{molec})(2.16\times 10^{21}\ \mathrm{molec/cm}^2)\right]^{1/2} - 1\right)\right]$$

$$\bar{\mathcal{T}}(u) = 0.630$$

The Malkmus band model evaluated for all three absorber amounts and compared to the mean transmission calculated with LBLRTM is

u (molec/cm^2)	$\mathcal{T}_{\Delta\nu}$ Malkmus	$\mathcal{T}_{\Delta\nu}$ LBLRTM
2.16×10^{20}	0.875	0.879
2.16×10^{21}	0.630	0.646
2.16×10^{22}	0.222	0.165

The band model transmissions are fairly close to the line-by-line model transmissions. Band models are more accurate for narrower bands where the line strength doesn't vary so much across the band. Band models can also be accurate if the band model parameters are obtained from fitting line-by-line model transmission rather than directly from the absorption line parameters.