Examples Week 13

1. *Calculate the solar zenith angle in Boulder at 3:00 pm on November 15.*

   The solar zenith angle is found from

   $$\cos\theta_0 = \sin\phi\sin\delta + \cos\phi\cos\delta\cos h$$

   where ϕ is the latitude, δ is the solar declination, and h is the hour angle.

   The solar declination is found from

   $$\sin\delta = \sin\epsilon\sin\lambda$$

   where λ is the longitude of the Earth from the vernal equinox. We'll assume the orbit is close enough to circular that we can use the linear in time approximation:

   $$\lambda = 360^\circ\frac{t_v}{T}$$

   where t_v is the time from the vernal equinox and T is the length of the year. November 15 is about 240 days since the vernal equinox, so

   $$\lambda = 360(240/365.24) = 236.6^\circ$$

   The Earth's tilt is $\epsilon = 23.5$, so the solar declination on November 15 is

   $$\sin\delta = \sin(23.5^\circ)\sin(236.6^\circ) = -0.3327 \qquad \delta = -19.4^\circ$$

   The latitude of Boulder is $\phi = 40^\circ$. We'll assume the longitude is -105° (center of the time zone) so that the hour angle is $h = 360^\circ(3\text{ hr}/24\text{ hr}) = 45^\circ$. Therefore, the solar zenith angle is

   $$\cos\theta_0 = \sin(40)\sin(-19.4) + \cos(40)\cos(-19.4)\cos(45) = 0.297 \quad \theta_0 = 72.7^\circ$$

2. *Calculate the day length in Boulder on November 15.*

   To find the day length we use the solar declination and the latitude:

   $$\cos H = -\tan\phi\tan\delta = -(0.839)(-0.353) = 0.296$$

   $H = 1.27\text{rad} = 72.8^\circ$. Day length is $(24\text{ hr})H/180^\circ = 9.7\text{ hrs}$

3. *Calculate the daily averaged TOA solar flux on November 15 in Boulder.*

November 15 is 49 days before perihelion, so the true anomaly is approximately

$$\nu \approx 360(-49/365.24) = -48.3^\circ$$

The ratio of the Earth-Sun distance to the mean distance is then

$$\frac{r}{r_0} \approx 1 - e\cos\nu = 1 - (0.017)(0.665) = 0.989$$

The daily averaged solar flux is

$$\bar{F} = \frac{S_0}{\pi}\left(\frac{r_0}{r}\right)^2 (H\sin\phi\sin\delta + \sin H\cos\phi\cos\delta)$$

Need $H = 1.27$ in radians (but be careful with $\sin H$)

$$\bar{F} = S_0/\pi \left(\frac{r_0}{r}\right)^2 [1.27\sin 40\sin(-19.4) + \sin 72.8\cos 40\cos(-19.4)]$$

$$\bar{F} = S_0(0.989)^{-2}(-0.272 + 0.690)/\pi = S_0(1.022)(.133)$$

$$\bar{F} = (1366\ \mathrm{W/m^2})(0.136) = 186\ \mathrm{W/m^2}$$