Examples Week 1

1. Suppose an instrument orbiting at 500 km altitude is viewing a dark ocean scene towards nadir in which the radiance is 50 $\mathrm{W\ m^{-2}\ sr^{-1}}\mu\mathrm{m}^{-1}$ at 0.5 μm. The instrument's telescope focusses an 10×10 cm area on the primary mirror down to a 1000x1000 element CCD detector. The telescope optics gives a pixel size of 250 m on the Earth. There is a 100 nm spectral filter at the 0.5 μm wavelength before the CCD detector. How many photons does each CCD pixel accumulate in a 10 ms exposure?

First, how much energy goes into one CCD pixel during exposure?

$$E = I_\lambda\ \Delta\lambda\ A\ \Omega\ \Delta t$$

Solid angle:

$$\Omega = (\Delta x/R)^2\ = (250\ \mathrm{m}/500,000\ \mathrm{m})^2 = 2.5 \times 10^{-7}\ \mathrm{sr}$$

where Δx is resolution on Earth and R is distance to Earth.
Collecting area for one CCD pixel:

$$A = \left(\frac{10\ \mathrm{cm}}{1000}\right)^2 = 10^{-8}\ \mathrm{m}^2$$

Energy on one CCD pixel during exposure is:

$$E = (50\ \mathrm{W\ m^{-2}\ sr^{-1}}\mu\mathrm{m}^{-1})(2.5 \times 10^{-7}\ \mathrm{sr})(10^{-8}\ \mathrm{m}^2)(0.100\ \mu\mathrm{m})(0.01\ \mathrm{s})$$

$$E = 1.25 \times 10^{-16}\ \mathrm{J}$$

Second, what is the energy of a photon at 0.5 μm:

$$E_{photon} = \frac{hc}{\lambda}\ = \frac{(6.626 \times 10^{-34}\ \mathrm{J\ s})(2.998 \times 10^{8}\ \mathrm{m/s})}{0.5 \times 10^{-6}\ \mathrm{m}} = 3.97 \times 10^{-19}\ \mathrm{J}$$

Thus, there are about $(1.25 \times 10^{-16}\ \mathrm{J}/3.97 \times 10^{-19}\ \mathrm{J}) = 315$ photons per pixel. Is this enough?